Tatiana Babicheva is an inspiring teacher who believes that teaching is not just about imparting knowledge but also about being a lifelong learner.

This passion for growth and discovery led her to earn two PhDs: one in applied mathematics and its applications in Russia, and another in computer science from Université Paris-Saclay in France. She has published 18 books in Russian, English and French in the field of popular science, including mathematics at the competition level.

COMPETITION MATH FOR KIDS: THINK, COUNT, SOLVE!

PART 1

TATIANA BABICHEVA

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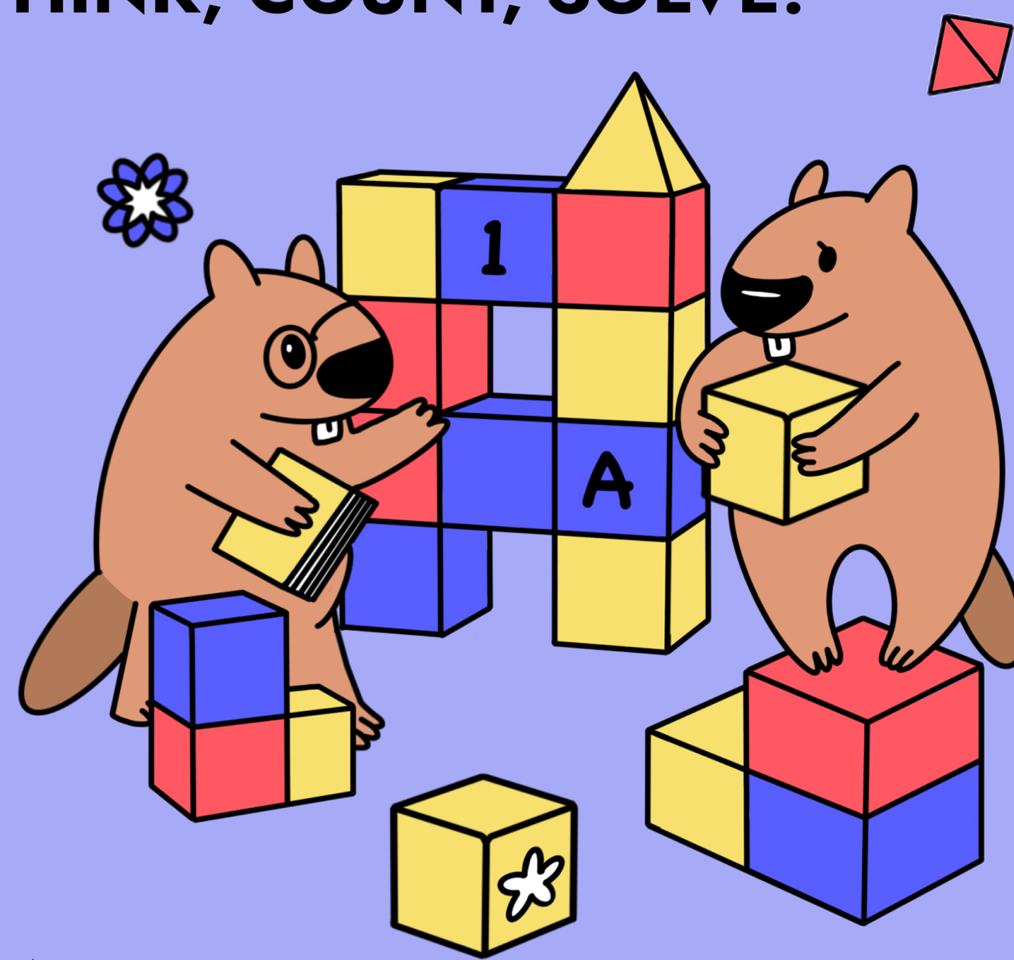


PART 1

Competition Math for Kids: Think, Count, Solve! Part 1

Illustrated by Yulia Chaika

Dedication

“ To my daughter Esther and my nephew Jean. I hope you will love mathematics as much as we do.

“ To our students, who inspired us to write this book. And to our main source of inspiration — Misha Bespalenko.

Introduction

Dear readers! If you are holding this book, it means you are just beginning your journey into competition mathematics.

We assume that:

1. You can read!
2. Counting up to 20 poses no difficulty for you (but, in reality, you are quite good at counting up to 100).
3. You know that the order of addends does not change the sum.
4. You can solve simple problems in one or two steps.
5. You can (or want to learn to) think!

We have listed the essential skills a budding math enthusiast needs. You can imagine this as a set of modeling clay, from which, by the end of this book, we will sculpt something new and beautiful.

Contents

Sequences

Theory and Practice

Our lives are filled with certain rules or, in other words, **patterns**. You already know how to count, so now you will likely be able to continue many sequences. For example, let's try solving the following problem.

Example 1.1. Continue the sequence: $1, 2, 3, 4\ldots$

Solution: This sequence is a progression of natural numbers, where each term is one greater than the previous one. In other words, we simply need to name the next numbers in order.

Answer: $1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\ldots$ □

So many new and unfamiliar words, right?

What is a **sequence**? In reality, it's almost the same as a series. A sequence is a series that follows a specific rule. For instance, the sequence in the example has a very simple rule — the numbers just follow in order.

By the way, a number in a sequence can be referred to in various ways: as an "element of the sequence," "term of the sequence," "term of the series," and more. We hope you won't get confused with all these *terms*.

What are **natural numbers**? Most likely, you haven't learned about other types of numbers yet. Natural numbers are the numbers we use to count things, such as the number of students in a class. These are numbers like $1, 2, 3 \ldots$ In other words, they are whole numbers greater than zero. Zero is a whole number but not a natural number.

There are also numbers called **negative numbers**. These are numbers less than zero. For example, in winter, if snow falls, the temperature often becomes negative.

You've probably also heard of **fractional numbers**. These are special non-integer numbers, such as "a half." Together, fractional and whole numbers make up **rational numbers**. You'll hear more about this term in eighth grade.

The rules by which sequences are formed can be very diverse. Let's solve another problem as an example.

Example 1.2. Identify the pattern (rule by which the numbers are written). Using this rule, continue the sequence:

$15, 14, 13, 12, 11 \ldots$

Solution: In this sequence, each number is one less than the previous one, which means it is "counting backwards." Therefore, the sequence should continue as follows:

Answer: $15, 14, 13, 12, 11, 10, 9, 8 \ldots$

What happens to this sequence when it reaches 0? If you haven't yet studied negative numbers, you'll have to stop at zero. □

Rules may also require some knowledge of mathematics. For example, it's important to know how to add and subtract. Let's solve the following problem.

Example 1.3. Continue the sequence: $1, 4, 7, 10 \ldots$

Solution: In this sequence, the difference between consecutive terms (subtracting the left number from the right) is 3. Let's verify: $4 - 1 = 7 - 4 = 10 - 7 = 3$. This means each successive term in the sequence is 3 greater than the previous one. Thus, after 10, the next term is $10 + 3 = 13$, followed by $13 + 3 = 16$, $16 + 3 = 19$, and so on.

Answer: $1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31 \dots$ □

What if we try something trickier? For instance, the operations could alternate.

Example 1.4. Continue the sequence: $1, 4, 3, 6, 5, 8, 7 \dots$

Solution: What is happening here? First, we added 3, then subtracted 1, then added 3 again, then subtracted 1, and so on. It seems we've identified the pattern! By continuing these operations, we get the following answer:

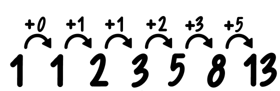
Answer: $1, 4, 3, 6, 5, 8, 7, 10, 9, 12, 11 \dots$ □

Sometimes, the pattern can be even more complex: we need to understand not only what is being added or subtracted each time but also consider the previous terms in the sequence.

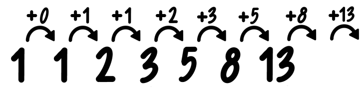
Let's look at one of the most famous sequences, known as the "**Fibonacci numbers**".

Example 1.5. Continue the sequence: $1, 1, 2, 3, 5, 8, 13 \dots$

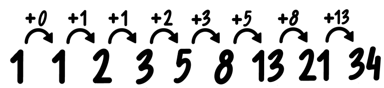
Solution: How can we identify such a pattern? The easiest way is to analyze the relationship between each pair of consecutive numbers.



What can we observe? The numbers written above essentially replicate those written below — if we ignore the initial zero. What happens next? Let's first extend the upper row so that it continues to mirror the lower row.



Now, knowing how the numbers progress, we can extend the lower row as well.



Answer: $1, 1, 2, 3, 5, 8, 13, 21, 34 \ldots$ □

Great, we've identified an interesting pattern and solved the problem. But why are the Fibonacci numbers so famous?

We could have approached this differently:

- $1 + 1 = 2$
- $1 + 2 = 3$
- $2 + 3 = 5$
- $3 + 5 = 8$
- $5 + 8 = 13$

We see that each new number in our sequence is the sum of the two preceding numbers. Isn't that a much more elegant explanation?

Speaking more "mathematically," Fibonacci numbers form a sequence that begins with 1 and 1, and each subsequent term is the sum of the two preceding ones. This creates the sequence $1, 1, 2, 3, 5, \ldots$

By the way, Fibonacci's real name wasn't Fibonacci at all. His name was Leonardo of Pisa, and he lived around 1170—1250, during the Middle Ages. As you might guess, he resided in the city of Pisa — the capital of the then Pisan Republic. Today, Pisa is part of Italy.

Interestingly, Leonardo of Pisa was one of the scholars who popularized the so-called Arabic numerals in Europe — the numbers we use today. Before that, people primarily used Roman numerals, which we'll discuss later.

Returning to patterns: Sometimes, instead of continuing a sequence, we need to find the missing numbers.

Example 1.6. Find the pattern in the sequence and insert the missing numbers: $18, 16, 14, \ldots, 10, 8, \ldots, 4, 2$.
Solution: In this sequence, the difference between consecutive terms is 2. Indeed:

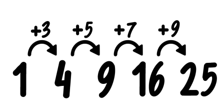
$$18 - 16 = 16 - 14 = 10 - 8 = 4 - 2 = 2.$$

This means that each subsequent term is 2 less than the previous one. The complete sequence will be: $18, 16, 14, \mathbf{12}, 10, 8, \mathbf{6}, 4, 2$.

Be sure to check that the inserted numbers do not break the pattern! Here, everything is correct: $14 - 12 = 12 - 10 = 8 - 6 = 6 - 4 = 2$. However, in competitions, especially in the future, you might encounter trickier questions, so it's important to stay vigilant. □

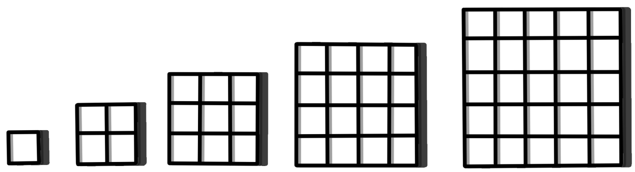
We've already discussed one sequence with a special name, but there are many such sequences.

Example 1.7. Continue the sequence: $1, 4, 9, 16, 25 \ldots$
Solution: As before, let's examine what happens between consecutive terms.



What do we notice? The numbers above form their own sequence. In this upper sequence, each subsequent number is 2 greater than the previous one. Thus, we should add $9 + 2 = 11$, $11 + 2 = 13$, and so on. Therefore, the next terms in the sequence will be $25 + 11 = 36$, $36 + 13 = 49$. □

We've found the solution to the problem. But, as with the Fibonacci numbers, there's more to the story. The numbers we've identified have a special name: they are called **square numbers**. Why are they called square numbers?

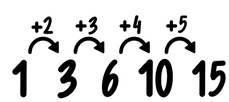


Let's draw squares. The first square has one cell, the second has four cells, the third has nine cells, the fourth has sixteen cells, and the fifth has twenty-five. Familiar numbers, aren't they? That's why these numbers are called square numbers: they represent the number of cells in successive squares.

Finally, let's make one more interesting observation. If you carefully examine the upper sequence, you'll notice that it consists of consecutive odd numbers (we'll discuss even and odd numbers in more detail later). It turns out that each square number is the sum of consecutive odd numbers. Fascinating, isn't it?

Example 1.8. Continue the sequence: $1, 3, 6, 10, 15 \ldots$

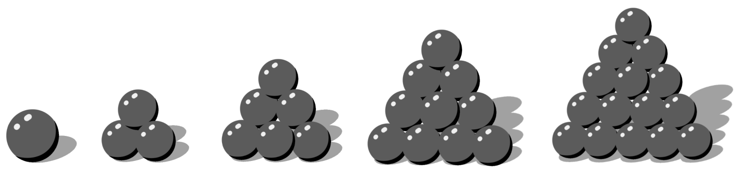
Solution: As before, let's analyze what happens between consecutive terms.



What do we notice? The numbers above also form a sequence, and it's the simplest kind: each subsequent number is 1 greater than the previous one. This means that to find the next term of the main sequence, we should add $5 + 1 = 6$, $6 + 1 = 7$, and so on. The next terms of our sequence will therefore be $15 + 6 = 21$, $21 + 7 = 28$. □

These numbers also have a special name: they are called **triangular numbers**. Why are they triangular?

Imagine stacking cannonballs into pyramids, or more precisely, into triangles. We observe the following pattern:



In the first pile, there's one cannonball; in the second pile, there are three; in the third, six; and so on. And yes, this is exactly the same sequence we just analyzed.

Let's tackle one last tricky problem that, at first glance, doesn't seem to have much to do with mathematics.

Example 1.9. Continue the sequence: O, T, T, F, F, S...

Solution: We might initially try to recall the alphabet, match each letter with its position, and so on. But that approach won't work here.

What do these letters actually mean? Let's think about counting in order. One, two, three, four, five... Now, what are the first letters of these words? **O**ne, **T**wo, **T**hree, **F**our, **F**ive. That's our sequence! Let's continue: **S**ix, **S**even, **E**ight, **N**ine...

Our answer is: O, T, T, F, F, S, S, E, N... □

We've explored many different sequences, which hopefully have made the topic less intimidating. You can now practice independently by working through additional exercises.

Practice Problems

Identify the pattern (rule for generating the numbers) and continue the given sequences.

Problem 1.1. $17, 19, 21, 23, 25\ldots$

Problem 1.2. $1, 2, 4, 8, 16\ldots$

Problem 1.3. $1, 2, 4, 7, 11\ldots$

Problem 1.4. $10, 6, 8, 4, 6\ldots$

Problem 1.5. $23, 23, 21, 25, 19, 27\ldots$

Problem 1.6. $1, 3, 5, 9, 15, 25\ldots$

For each sequence, identify the pattern and fill in the missing numbers:

Problem 1.7. $3, 6, \ldots, 12, 15, 18.$

Problem 1.8. $65, 60, 55, \ldots, 45, 40, 35.$

Problem 1.9. $20, \ldots, 21, 15, 22, 14, 23, 13.$

Problem 1.10. $45, 50, 40, \ldots, 35, 70, 30, 80.$

Problem 1.11. $2, 1, 3, 2, 4, 3, \ldots, 4, 6.$

Problem 1.12. $45, 36, 28, 21, \ldots, 10, 6.$

Solutions to Practice Problems

Solution to Problem 1.1: What is the pattern here? We add 2 every time. Thus, the sequence continues as:

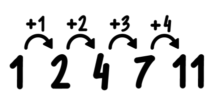
$17, 19, 21, 23, 25, 27, 29, 31 \ldots$ □

Solution to Problem 1.2: What happens here? $1 + 1 = 2$, $2 + 2 = 4$, $4 + 4 = 8$, $8 + 8 = 16 \ldots$

Each number is double the previous one. Continuing, we get: $16+16 = 32$, $32+32 = 64$, $64 + 64 = 128$.

Answer: $1, 2, 4, 8, 16, 32, 64, 128 \ldots$ □

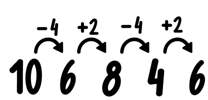
Solution to Problem 1.3: Let's examine the differences between consecutive terms:



We observe that each time, the number added increases by 1. Thus, we should add 5, 6, and so on. After 11, we get $11 + 5 = 16$, followed by $16 + 6 = 22$.

Answer: $1, 2, 4, 7, 11, 16, 22, 29 \ldots$ □

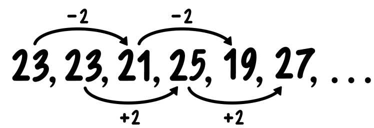
Solution to Problem 1.4: Let's analyze the differences between consecutive terms:



First, we subtract 4, then add 2, then subtract 4 again, and so on. Continuing this alternating pattern, after 6, we have $6 - 4 = 2$, then $2 + 2 = 4$, and so on.

Answer: $10, 6, 8, 4, 6, 2, 4, 0 \ldots$ □

Solution to Problem 1.5: This one is trickier! We need to examine the numbers at every other position. Let's write them out:



The next number at every other position after 19 is $19 - 2 = 17$. The number after 27 is $27 + 2 = 29$, and so on.

Answer: $23, 23, 21, 25, 19, 27, 17, 29, 15, 31 \ldots$ $\square$

Solution to Problem 1.6: This is the most complex sequence in the tasks.

- $1 + 3 = 4$, and if we add 1 more, it becomes $4 + 1 = 5$;
- $3 + 5 = 8$, and $8 + 1 = 9$;
- $5 + 9 = 14$, and $14 + 1 = 15$;
- $9 + 15 = 24$, and $24 + 1 = 25$.

Thus, we add the previous two numbers and then add 1. What happens next?

- $15 + 25 = 40$, and $40 + 1 = 41$;
- $25 + 41 = 66$, and $66 + 1 = 67$.

Answer: $1, 3, 5, 9, 15, 25, 41, 67 \ldots$ $\square$

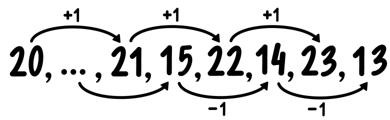
Solution to Problem 1.7: Here, each number increases by 3. Thus, the missing number is $6 + 3 = 9$.

Answer: $3, 6, \mathbf{9}, 12, 15, 18.$ $\square$

Solution to Problem 1.8: Here, each number decreases by 5. Thus, the missing number is $55 - 5 = 50$.

Answer: $65, 60, 55, \mathbf{50}, 45, 40, 35.$ $\square$

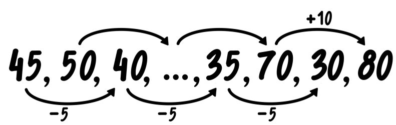
Solution to Problem 1.9: This is another sequence where we look at alternating numbers:



The missing number is 1 more than 15.

Answer: 20, **16**, 21, 15, 22, 14, 23, 13. □

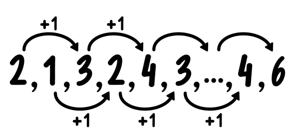
Solution to Problem 1.10: Here, we again examine numbers at every other position:



The missing number is 10 more than 50 or, equivalently, 10 less than 70. This number is 60.

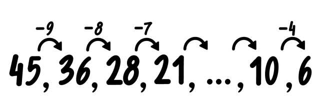
Answer: 45, 50, 40, **60**, 35, 70, 30, 80. □

Solution to Problem 1.11: Here, let's examine numbers at every other position:



Answer: 2, 1, 3, 2, 4, 3, **5**, 4, 6. □

Solution to Problem 1.12: Here, we note the differences between consecutive numbers:



In the upper sequence, the difference decreases by 1 each time. After subtracting 7, the next step is to subtract 6, and then 5. Thus, the missing number is $21 - 6 = 15$.

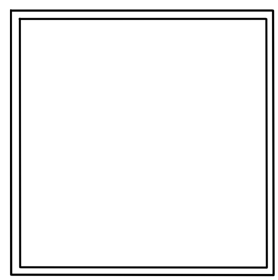
Answer: 45, 36, 28, 21, **15**, 10, 6. □

Chairs and Walls

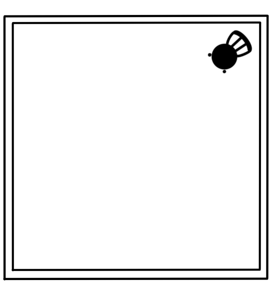
Theory and Practice

Let's start with a classic Olympiad problem.

Example 2.1. Arrange 5 chairs in a square room such that each of the four walls has exactly 2 chairs next to it. Chairs must be placed only along the walls, not in the center of the room!

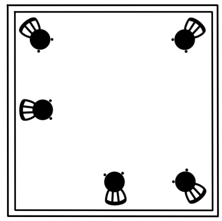


Solution: At first glance, it seems impossible—how can you divide 5 chairs evenly among 4 walls?



Additionally, 4 walls with 2 chairs each would require $4 \times 2 = 8$ chairs, not 5. But there's a trick: if you place a chair in a corner, it counts as being next to two walls simultaneously. For instance, in the diagram on the left, the chair is next to both the top and the right walls.

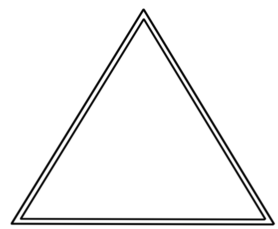
By experimenting with different configurations, you can achieve this arrangement:



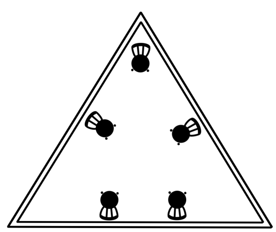
□

Rooms, of course, don't always have four walls. For example, some rooms are triangular.

Example 2.2. Arrange 5 chairs in a triangular room such that each of the three walls has exactly 2 chairs next to it. Chairs must be placed only along the walls, not in the center of the room!



Solution: After trying a few configurations, you can achieve the following arrangement:



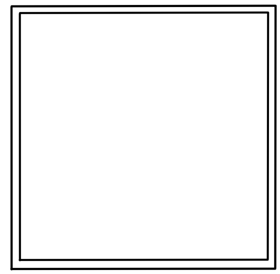
□

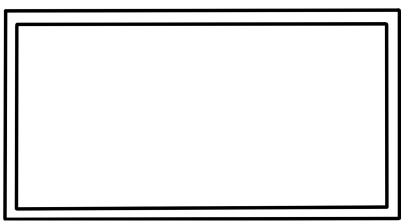
To simplify the problem, consider the number of chairs that must be placed in corners (thus belonging to two walls). If no chairs were placed in corners, then to satisfy the requirement of 2 chairs per wall, we would need $2 + 2 + 2 = 6$ chairs. However, we only have 5 chairs in total. This means $6 - 5 = 1$ chair must be placed in a corner and count for two walls.

In this chapter, we don't introduce much new theory. The key is to remember to check the corners and carefully count the number of chairs next to each wall. Now, practice solving these challenges at home!

Practice Problems

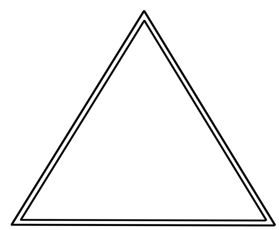
Problem 2.1. Show how to arrange 6 chairs in a square room so that each of the four walls has exactly 2 chairs next to it. Chairs must be placed only along the walls and not in the center!

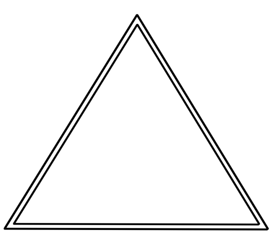




Problem 2.2. Show how to arrange 14 chairs in a square room so that all four walls have the same number of chairs. Chairs must be placed only along the walls and not in the center!

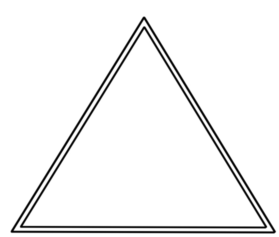
Problem 2.3. Show how to arrange 10 chairs in a triangular room so that each of the three walls has the same number of chairs. Chairs must be placed only along the walls and not in the center!



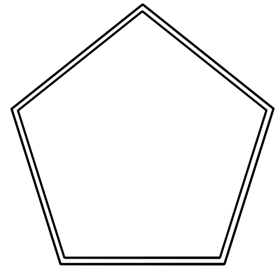


Problem 2.4. Show how to arrange 11 chairs in a triangular room so that each of the three walls has the same number of chairs. Chairs must be placed only along the walls and not in the center!

Problem 2.5. Show how to arrange 9 chairs in a triangular room so that each of the three walls has the same number of chairs. Chairs must be placed only along the walls and not in the center!

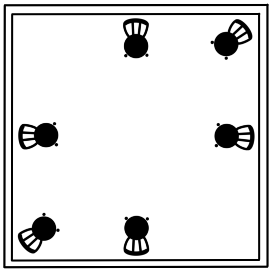


Problem 2.6. Show how to arrange 9 chairs in a pentagonal room so that each of the walls has the same number of chairs. Chairs must be placed only along the walls and not in the center!



Solutions to Practice Problems

Solution to Problem 2.1:



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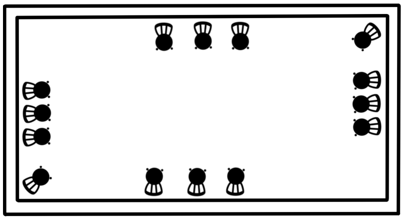
Solution to Problem 2.2:



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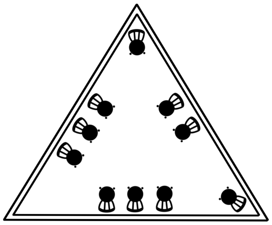
Solution to Problem 2.3:



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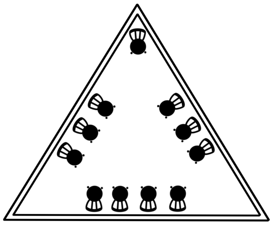
Solution to Problem 2.4:



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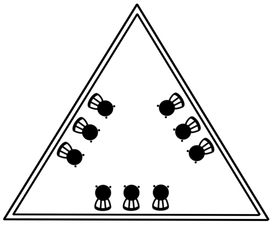
Solution to Problem 2.5:



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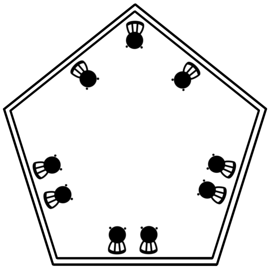
Solution to Problem 2.6:



□

What's Missing?

Theory and Practice

You've probably encountered problems where you need to fill in a missing number. As you progress to higher grades, you'll meet this topic again, but under its real name — **equations**.

Let's start with three foundational examples on this topic.

Example 3.1. What number should replace the $\triangle$?

$$2 + \triangle = 5.$$

Solution: Of course, we can find the missing number using the simple trial and error method. But you might already know a useful rule: *To find an addend, subtract the other addend from the sum.* Using this rule:

$$\triangle = 5 - 2,$$

$$\triangle = 3.$$

Don't forget to check your work! Substitute 3 for the triangle: $2+3 = 5$. This is true, which means our solution is correct. $\square$

When solving subtraction problems, it's important to understand terms like minuend, subtrahend, and difference.

Minuend — the number from which something is subtracted.

Subtrahend — the number that is subtracted from the minuend.

Difference — the result after subtraction.

For example, in $16 - 7 = 9$, the number 16 is the minuend because we subtract from it. The number 7 is the subtrahend because it's subtracted from 16. The number 9 is the difference — the result of subtracting 7 from 16.

Example 3.2. What number should replace the $\triangle$?

$$7 - \triangle = 5.$$

Solution: Here we can use another rule: *To find the subtrahend, subtract the difference from the minuend.* Applying this rule:

$$\triangle = 7 - 5,$$

$$\triangle = 2.$$

Check: Substitute 2 for the triangle: $7 - 2 = 5$. This is true, so the triangle is indeed equal to 2. $\square$

Example 3.3. What number should replace the $\triangle$?

$$\triangle - 3 = 5.$$

Solution: In this problem, we use another rule: *To find the minuend, add the subtrahend and the difference.* Applying this rule:

$$\triangle = 5 + 3,$$

$$\triangle = 8.$$

Check: Substitute 8 for the triangle: $8 - 3 = 5$. This is true, so the triangle is indeed 8. $\square$

Of course, these problems are very simple. But the same logic can be applied to solve more complex problems: when solving multi-step problems, you just need to perform the steps in the correct order.

Example 3.4. What number should replace the $\triangle$?

$$2 + \triangle + 6 = 10.$$

Solution: First, we figure out the value of the expression $2 + \triangle$.

$$\boxed{2 + \triangle} + 6 = 10.$$

To do this, remember the rule: *To find an addend, subtract the other addend from the sum.* Thus, $2 + \triangle = 10 - 6 = 4$. Using the same logic for the new expression, we find $\triangle = 4 - 2 = 2$.

Check:

$$2 + 2 + 6 = 10,$$

$$10 = 10.$$

Everything checks out, so we're correct. □

Solution, using rules: We hope you've already learned that *changing the order of addends does not change their sum.* What does this mean in our case?

It means we can rewrite our condition like this:

$$2 + \triangle + 6 = 10,$$

$$2 + 6 + \triangle = 10.$$

Then, by performing the first step (adding 2 and 6), we get:

$$8 + \triangle = 10,$$

From which we find:

$$\triangle = 10 - 8,$$

$$\triangle = 2.$$

We already verified this solution with the previous method and know that 2 is the correct answer. □

Which of these two solutions is simpler? That depends on you. The second method might lead to larger numbers, while the first requires a bit more attentiveness. We encourage you to master both methods now, as in school you might not always have a choice.

Example 3.5. What number should replace the $\square$?

$$7 + \square - 4 = 10$$

Solution: First, swap the positions of two addends, such as 7 and $\square$. This gives us $\square + 7 - 4 = 10$. Notice that $7 - 4$ equals 3. Let's perform this calculation first.
This simplifies to $\square + 3 = 10$.
Then, *move* 3 to the other side, which gives us $\square = 10 - 3$,
$\square = 7$.

This clever step, called *rearranging*, will be discussed in detail in the chapter on balance scales. Formally, we simply subtracted 3 from both sides of the equation.

Since there were several steps and reordering, don't forget to verify:

$$7 + 7 - 4 = 10,$$

$$10 = 10.$$

Everything checks out, so we got it right. $\square$

Example 3.6. What number should replace the $\bigcirc$?

$$12 - \bigcirc + 1 = 6$$

Solution: First, find the value of the expression $12 - \bigcirc$. To do this, recall that you can find an addend by subtracting the other addend from the sum. Thus, $12 - \bigcirc = 6 - 1 = 5$.

Now, applying the same logic as the second example, we find:

$$\bigcirc = 12 - 5,$$

$$\bigcirc = 7.$$

Verification:

$$12 - 7 + 1 = 6,$$
$$6 = 6.$$

Everything checks out, and the problem is solved correctly. □

Practice Problems

Problem 3.1. What number should replace the $\bigcirc$?

$$8 + \bigcirc - 5 = 10.$$

Problem 3.2. What number should replace the $\bigcirc$?

$$20 - \bigcirc + 2 = 10.$$

Problem 3.3. What number should replace the $\bigcirc$?

$$1 + 2 + 5 + \bigcirc + 6 = 30.$$

Problem 3.4. What number should replace the $\bigcirc$?

$$20 - \bigcirc + 5 - 4 = 10.$$

Problem 3.5. What number should replace the $\bigcirc$?

$$\bigcirc - 5 + 10 + 2 + 1 = 20.$$

Solutions to Practice Problems

For all the tasks in this section, we provide brief solutions without verification. We encourage you to complete the verification on your own!

Solution to Problem 3.1:

$$8 + \bigcirc = 10 + 5,$$

$$8 + \bigcirc = 15,$$

$$\bigcirc = 15 - 8,$$

$$\bigcirc = 7.$$

□

Solution to Problem 3.2:

$$20 - \bigcirc = 10 - 2,$$

$$20 - \bigcirc = 8,$$

$$\bigcirc = 20 - 8,$$

$$\bigcirc = 12.$$

□

Solution to Problem 3.3:

$$1 + 2 + 5 + \bigcirc = 30 - 6,$$

$$1 + 2 + 5 + \bigcirc = 24,$$

$$3 + 5 + \bigcirc = 24,$$

$$8 + \bigcirc = 24,$$

$$\bigcirc = 24 - 8,$$

$$\bigcirc = 16.$$

□

Solution to Problem 3.4:

$$20 - \bigcirc + 5 = 10 + 4,$$

$$20 - \bigcirc + 5 = 14,$$
$$20 - \bigcirc = 14 - 5,$$
$$20 - \bigcirc = 9,$$
$$\bigcirc = 20 - 9,$$
$$\bigcirc = 11.$$

□

Solution to Problem 3.5:

$$\bigcirc - 5 + 10 + 2 = 20 - 1,$$

$$\bigcirc - 5 + 10 + 2 = 19,$$
$$\bigcirc - 5 + 10 = 19 - 2,$$
$$\bigcirc - 5 + 10 = 17,$$
$$\bigcirc - 5 = 17 - 10,$$
$$\bigcirc - 5 = 7,$$
$$\bigcirc = 7 + 5,$$
$$\bigcirc = 12.$$

□

Logic: Negations

Theory and Practice

In many school-level math Olympiads, problems often fall into a general category of so-called "logic problems."

What exactly is **logic**? Essentially, it's just reasonable reasoning. But doesn't that make every problem a logic problem? Well, in a way, yes. However, when we talk about logic problems, we generally mean problems where reasoning is more important than computation. For such problems, it is crucial to know how to construct negations, i.e., to "say the opposite."

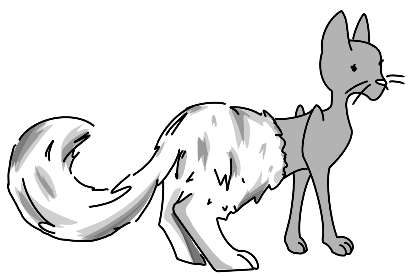
In mathematics, ”the opposite” is often referred to as **falsehood**.

First, let's learn to determine what is true and what is false. Identify whether the following statements are true or false:

The Earth orbits the Sun. Hopefully, you know this is true.
$\mathbf{2+9=10}$. This is false, as $2+9=11$.
Five is less than six. This is true.
Five is less than four. This is false.
Five is less than five. This is also false because five equals five, not less than it!
Grass is green. In a math problem, this is likely considered true, though exceptions can exist—for example, grass turns yellow in autumn.

Now let's practice **constructing negations**, i.e., saying something opposite. For example, the negation of truth is falsehood. Or vice versa, we can correct someone who gave us a false statement by saying the truth.

Statement: The Earth orbits the Sun. Negation: The Earth **does not** orbit the Sun.
Statement: $2 + 9 = 10$. Negation: $2 + 9$ **is not equal to** 10.
Statement: Five is less than six. Negation: Five is **not** less than six.
Statement: Grass is green. Negation: Grass is **not** green.
Statement: All cats are fluffy. Negation: **Not all** cats are fluffy.
Statement: Cows have two horns. Negation: Cows **do not** have two horns.



As you can see, constructing a negation can often be done by simply adding the word "not." The key is to place it in the correct spot, or the meaning of the sentence may change entirely. For instance, the phrase *all non-cats are fluffy* is technically a negation, but not of the intended statement. Imagine a world where everything that isn't a cat—whales, houses, and even you—becomes fluffy!

But what's wrong with the phrase *all cats are not fluffy*? Why is this not the correct negation? Think about it. Clearly, the statement is false because many cats are fluffy. However, the original statement that *all cats are fluffy* is also false. Why? Because some cats, like the Sphynx breed, are hairless.

So does this mean the negation of a falsehood is also false? Something went wrong here. In more advanced classes, you'll learn the laws of negation. For now, remember that the negation of "all" is "not all."

Practice Problems

Problem 4.1. For each of the phrases below, determine whether they are true or false or whether we cannot say them for sure.

- $25 + 17 = 43$.
- All beavers build dams.
- This problem is very difficult.
- The current year number is greater than 2023.
- $6 + 9$ is less than 18.

Problem 4.2. For each of the phrases below, construct its negation.

- The little fir tree is cold in winter.
- Everyone loves porridge.
- Time travel is already possible.
- The author of this book believes that you will solve this problem incorrectly.
- 49 is less than 100.

Solutions to Practice Problems

Solution to Problem 4.1: $\mathbf{25 + 17 = 43.}$ This is false because $25 + 17 = 42$, not 43.

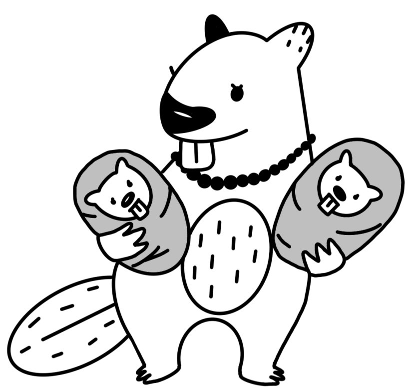
All beavers build dams. If the statement referred to all **adult** beavers, it would likely be true. However, this refers to **all** beavers, including the youngest ones. Therefore, it is false.

This problem is very difficult. Here, we cannot be sure. For some, this problem might be difficult, while for others (like a professor), it could be very easy.

The current year number is greater than 2023. We wrote this book in 2024, and 2024 is indeed greater than 2023. If you are not a time traveler, you are certainly reading this after 2023. Thus, this is true.

$\mathbf{6 + 9}$ **is less than 18.** Adding the numbers gives $6+9 = 15$. Since 15 is indeed less than 18, this is true.

□



Solution to Problem 4.2: **The little fir tree is cold in winter.** The little fir tree is **not** cold in winter.

Everyone loves porridge. Not everyone loves porridge.

Time travel is already possible. Time travel is **not yet possible.** This was a tricky one. Why? It might seem easy to just add "not" and say time travel is **already not possible.** However, that would imply time travel was possible at some point, which is not true. Thus, we had to replace "already" with "not yet" to maintain logical consistency.

The author of this book believes you will solve this problem incorrectly.
The author of this book **does not** believe you will solve this problem incorrectly.

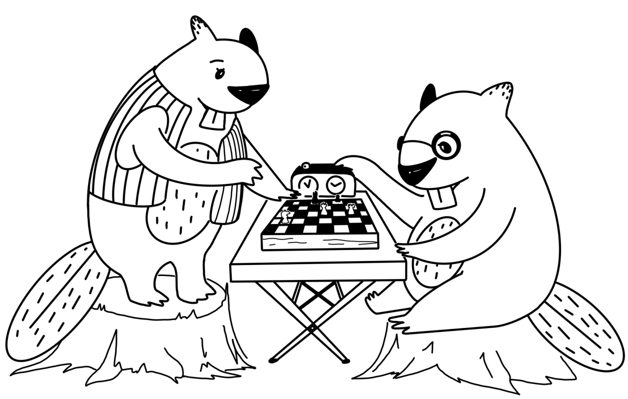
49 is less than 100. 49 is **not** less than 100.

□

Chess in Mathematics

Theory and Practice

Problems involving the chessboard and chess pieces are frequent guests at math olympiads. We will encounter the straightforward rook, which cannot be knocked off its horizontal or vertical path, the steadfast bishop, which refuses to move to a square of a different color, the versatile queen, which combines the powers of the rook and bishop, the cunning knight, which can jump over other pieces, and the slow-moving king, which cannot move more than one step in any direction.

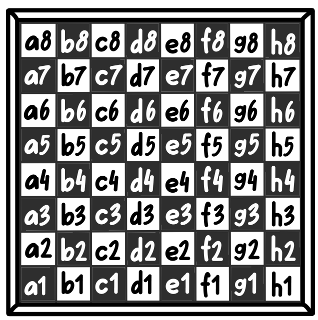


Chessboard-related problems are so common in olympiads that we cannot ignore this topic. Usually, the problem statement does not describe how the pieces move, but since knowledge of chess rules is not part of the standard school curriculum, the jury is required to answer any questions about the movement of pieces, as these rules are considered part of the problem conditions.

Unless stated otherwise, the chessboard is assumed to have dimensions of 8×8 squares.

The chessboard is colored in two colors — white and black — according to the following principle: if one square is white, all adjacent squares sharing an edge with it must be black, and vice versa. Thus, in any row or column, adjacent squares will have different colors, while squares separated by one square or touching at corners will have the same color. This pattern is called a "**checkerboard pattern**".

The squares of the chessboard are labeled with letters from a to h (left to right) and numbers from 1 to 8 (bottom to top). The squares are always colored such that $a1$ is a black square. Below is an illustration showing the labels around the board and the name of each square. Sometimes we use uppercase these letters, and sometimes lowercase; this does not matter.



According to legend, when the inventor of chess presented his creation to the king, the king was so impressed that he promised to fulfill any wish of the inventor in return for the invention.

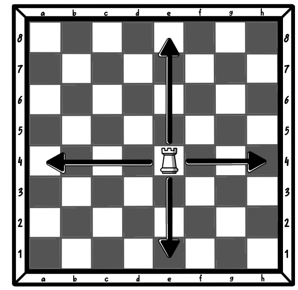
The inventor, being very clever but not very wise, made the following request: for the first square of the chessboard, he would receive one grain of rice; for the second square — two grains; for the third — four grains; for the fourth — eight grains, and so on. Sound familiar? This is a sequence where the number of grains doubles each time. Eventually, the king realized that this would require more rice than had ever been grown in the kingdom and punished the inventor.

You can try to calculate how much rice this would be. By the tenth square, you will likely already feel exhausted. For example, by the twentieth square, the number of grains will already exceed one million!

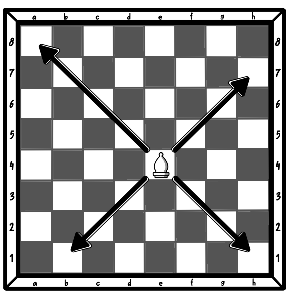
Let us recall the chess rules that may be useful for mathematical olympiads:

1. **Rook.**

   On its turn, the rook can move any number of squares either horizontally or vertically, without the ability to jump over other pieces. Interestingly, in German, the rook is called a “tower”. The diagram on the right illustrates possible rook moves.

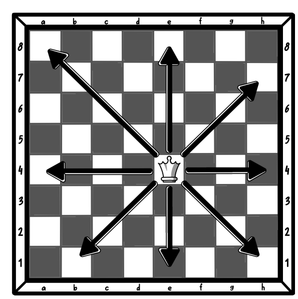
2. **Bishop.**

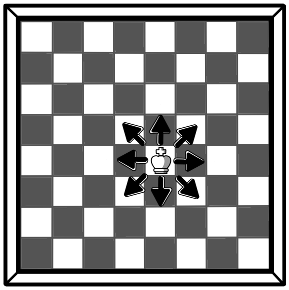
   The bishop moves any number of squares diagonally, also without the ability to jump. In other languages, it is referred to as a “runner” or “officer”. The diagram on the left illustrates possible bishop moves.

3. **Queen.**

   The queen combines the abilities of the rook and bishop: it can move any number of squares horizontally, vertically, or diagonally, without the ability to jump. The diagram illustrates possible queen moves.

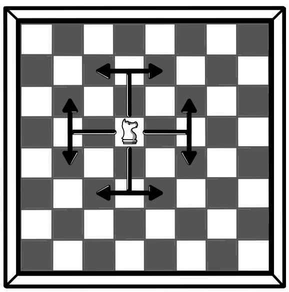
   

4. **King.**



The king moves one square in any direction: horizontally, vertically, or diagonally. In other words, it can move to any square that shares an edge or a vertex with its current square. The diagram illustrates possible king moves.

5. **Knight.**



The knight moves in the following way: its new position must differ from the initial position by two squares horizontally and one square vertically, or one square horizontally and two squares vertically.

For the knight's move to be valid, the destination square must be unoccupied by a piece of its own color. However, the presence of other pieces elsewhere on the board does not obstruct the knight's movement. You may have heard the phrase, "The knight moves in an 'L' shape". This describes the unique way the knight jumps. In German, it is called the "jumper", while in Russian, it is referred to as the "horse". The diagram illustrates possible knight moves.

Of course, the pieces must stay within the boundaries of the chessboard and cannot make a move beyond its edges.

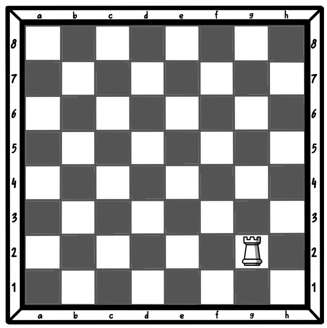
Typically, the phrase "make a move" implies moving to an empty square that is not occupied by another piece. Capturing (moving to a square already occupied by another piece) is a legal move in real chess games. However, in math olympiads, the possibility of capturing is usually specified explicitly.

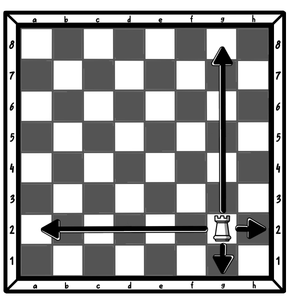
It is said that a piece A *attacks* a square b, or that square b is *under attack* by piece

A, if the piece can legally move to that square—even if another piece occupies it. For pawns, the definition is different due to their unique movement rules, but we will not encounter pawns in our current mathematical problems.

Let's practice identifying the squares attacked by a chess piece.

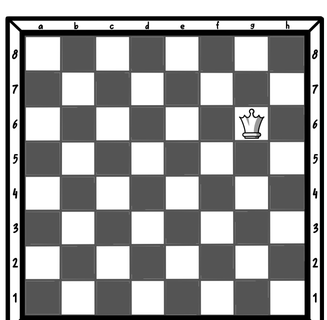
Example 5.1. Name the squares attacked by the piece on the chessboard.



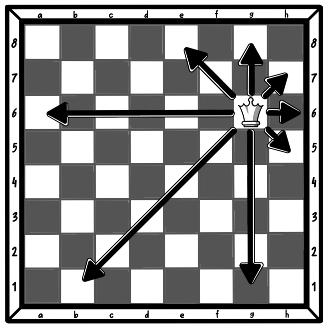


Solution: The board shows a rook positioned at square $g2$.
The rook attacks all squares along its vertical and horizontal lines. Let's mark them.
Now, let's list all the attacked squares: $a2$, $b2$, $c2$, $d2$, $e2$, $f2$, $h2$, $g1$, $g3$, $g4$, $g5$, $g6$, $g7$, $g8$.
Notice that we listed the squares by rows and columns in an orderly fashion to avoid confusion. □

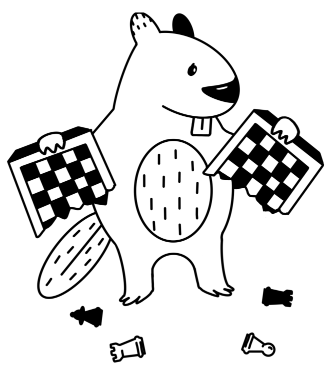
Example 5.2. Name the squares attacked by the piece on the chessboard.



Solution: The board shows a queen positioned at square $g6$. The queen attacks all squares along its vertical, horizontal, and diagonal lines. Let's highlight these squares:

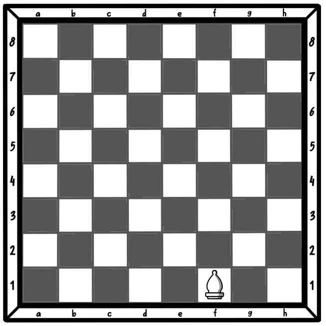


Listing all the squares under attack by the queen, line by line, we get: $a6$, $b6$, $c6$, $d6$, $e6$, $f6$, $h6$, $g1$, $g2$, $g3$, $g4$, $g5$, $g7$, $g8$, $b1$, $c2$, $d3$, $e4$, $f5$, $h7$, $e8$, $f7$, $h5$. $\square$

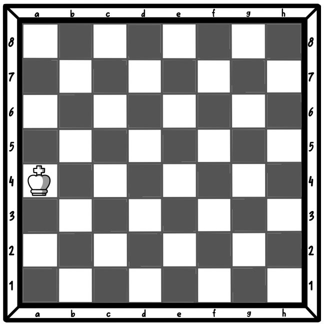


Practice Problems

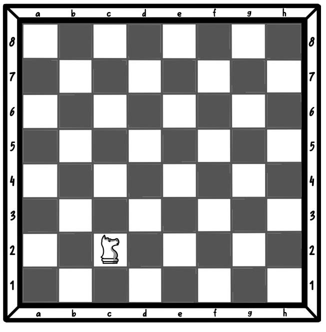
Problem 5.1. Name the squares attacked by the piece on the chessboard.



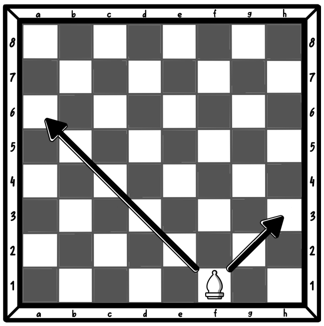
Problem 5.2. Name the squares attacked by the piece on the chessboard.



Problem 5.3. Name the squares attacked by the piece on the chessboard.

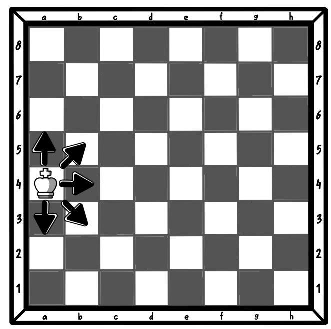


Solutions to Practice Problems

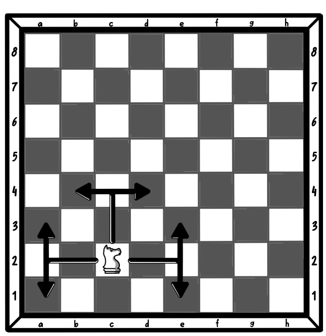


Solution to Problem 5.1: The piece on the board is a bishop positioned on square $f1$. The bishop attacks all squares along its diagonals. Let's mark these squares.
Enumerating the attacked squares along the diagonals, we get: $a6$, $b5$, $c4$, $d3$, $e2$, $g2$, $h3$. □

Solution to Problem 5.2: The piece on the board is a king positioned on square $a4$. The king attacks all adjacent squares. Let's mark these squares.
Enumerating the attacked squares clockwise, we get: $a5$, $b5$, $b4$, $b3$, $a3$. □



Solution to Problem 5.3: The piece on the board is a knight positioned on square $c2$. The knight attacks all squares it can reach with its "L-shaped" moves. Let's mark these squares:

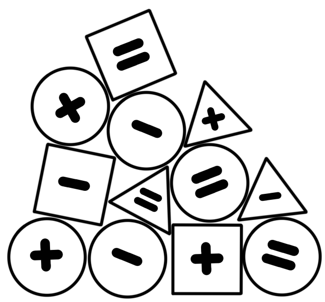


Enumerating the attacked squares clockwise, we get: $a1$, $a3$, $b4$, $d4$, $e3$, $e1$. □

Placing Arithmetic Signs

Theory and Practice

Previously, we learned how to determine which numbers are hidden behind symbols. But what if the symbols conceal arithmetic operations instead of numbers?



Example 6.1. Replace $\triangle$ with the signs $+$ and $-$ to make the equation true. The triangles may represent different signs.

$$5 \triangle 2 \triangle 1 = 6.$$

Solution: Let's try a few combinations. If we subtract 2 from 5, we get 3. Adding 1 results in 4, which is still too small. Therefore, we must add 2 first. Thus, $5 + 2 = 7$, and then subtract 1 to get 6. The answer is:

$$5 + 2 - 1 = 6.$$

□

Example 6.2. Replace $\triangle$ with the signs $+$ and $-$ to make the equation true:

$$20 \triangle 7 \triangle 4 = 17.$$

Solution: If we add 7 to 20, the result is 27. Subtracting 4 gives 23, which is too large. Therefore, we need to subtract 7 first: $20-7 = 13$. Adding 4 gives $13+4 = 17$. The answer is:

$$20 - 7 + 4 = 17.$$

$\square$

Not too hard, right? Now let's consider a problem with three signs.

Example 6.3. Replace $\triangle$ with the signs $+$ and $-$ to make the equation true:

$$3 \triangle 8 \triangle 4 \triangle 7 = 8.$$

Solution: Notice that the entire expression equals 8, and 8 is one of the terms on the left side of the equation.

To simplify, let's replace part of the equation with a single variable: assume $\square = 3 \triangle 4 \triangle 7$. Then the equation becomes $8 + \square = 8$. Since adding anything other than 0 would change the value, it's clear that $\square = 0$.

Now, let's calculate $3 \triangle 4 \triangle 7 = 0$. Since $3+4 = 7$, we find that $3+4-7 = 0$. Thus, the answer is:

$$3 + 8 + 4 - 7 = 8.$$

$\square$

Example 6.4. Replace $\square$ with the signs $+$ and $-$ to make the equation true:

$$3 \square 2 \square 9 \square 2 = 12.$$

Solution: Notice that $3+9 = 12$. To balance the equation, we need to place different signs in front of the remaining twos. Thus, we get:

$$3 - 2 + 9 + 2 = 12$$

or

$$3 + 2 + 9 - 2 = 12.$$

$\square$

Sometimes, even the equals sign can be "hidden."

Example 6.5. Replace $\triangle$ with the signs $+$, $-$, and $=$ to form a valid equation:

$$8 \triangle 3 \triangle 7 \triangle 2.$$

Solution: We can quickly recall that $8 - 3 = 5$ and $7 - 2 = 5$. Then, the signs can be arranged as follows:

$$8 - 3 = 7 - 2.$$

□

But what should we do when there are many signs to place? As always, we need to look for patterns and systematically try different possibilities. Such problems are typically encountered only in math competitions, where you usually have ample time to think and test various methods.

Example 6.6. Replace $\bigcirc$ with the signs $+$ and $-$ to make the equation true:

$$8 \bigcirc 6 \bigcirc 9 \bigcirc 3 \bigcirc 7 \bigcirc 2 = 17$$

Solution: If we carefully examine the numbers on the left, we notice 8 and 9, which together equal 17. Let's try adding them. The remaining numbers must sum to zero. After thorough testing—or perhaps a bit of luck—we find the answer:

$$8 - 6 + 9 - 3 + 7 + 2 = 17.$$

Don't forget to carefully check your calculations step by step to ensure the solution is correct! Let's do it together:

1. $8 - 6 = 2$,
2. $2 + 9 = 11$,
3. $11 - 3 = 8$,
4. $8 + 7 = 15$,
5. $15 + 2 = 17$.

□

Although we didn't explicitly verify each step in the previous problems, you should always perform such checks. However, since this is more of a school-level math routine than a competition-level technique, we decided not to burden you unnecessarily.

Practice Problems

Problem 6.1. Replace $\triangle$ with the signs $+$ and $-$ to make the equation true:

$$5 \triangle 6 \triangle 8 \triangle 7 = 12.$$

Problem 6.2. Replace $\triangle$ with the signs $+$ and $-$ to make the equation true:

$$3 \triangle 4 \triangle 5 \triangle 12 = 14.$$

Problem 6.3. Replace $\triangle$ with the signs $+$ and $-$ to make the equation true:

$$3 \triangle 14 \triangle 15 \triangle 9 \triangle 2 \triangle 6 = 15.$$

Problem 6.4. Replace $\triangle$ with the signs $+$, $-$, and $=$ to create a valid equation:

$$10 \triangle 3 \triangle 2 \triangle 5 \triangle 19 \triangle 8 \triangle 5.$$

Problem 6.5. Replace $\triangle$ with the signs $+$, $-$, and $=$ to create a valid equation:

$$1 \triangle 2 \triangle 3 \triangle 4 \triangle 5 \triangle 6 \triangle 7 \triangle 8 \triangle 2.$$

Solutions to Practice Problems

Solution to Problem 6.1: The solution is:

$$5 + 6 + 8 - 7 = 12.$$

□

Solution to Problem 6.2: The solution is:

$$3 + 4 - 5 + 12 = 14.$$

□

Solution to Problem 6.3: One possible solution is:

$$3 + 14 + 15 - 9 - 2 - 6 = 15.$$

□

Solution to Problem 6.4: One possible solution is:

$$10 + 3 - 2 - 5 = 19 - 8 - 5.$$

□

Solution to Problem 6.5: One possible solution is:

$$1 + 2 - 3 + 4 + 5 - 6 + 7 = 8 + 2.$$

□

Domino, Trimino, and Tetromino

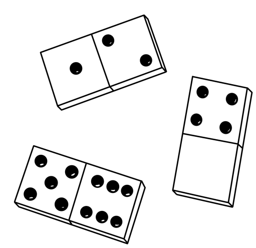
Theory and Practice

We have solved many problems involving numbers. Now let's consider a topic that is a bit closer to geometry.

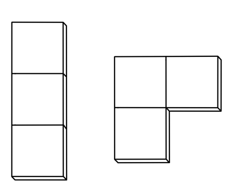
You have probably already encountered a game called **domino**.

Domino is a tabletop game that uses tiles with numbers ranging from 0 to 6. Each tile is divided into two parts, and each part contains a number. Players must place the tiles next to each other so that the numbers on the touching sides match.

Examples of some domino tiles are shown on the right.
If we ignore the dots on the domino tiles, we see that a **domino** is simply a figure made up of two squares. It can lie vertically or horizontally, but for now, its orientation is not important.



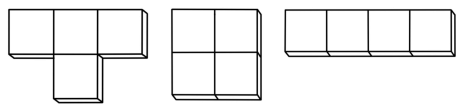
A **trimino** is a figure similar to a domino, but it has three parts instead of two. That is, a trimino is a figure made up of three squares, cut out from graph paper along the grid lines. Figures that can be made identical by rotation or reflection are considered the same. So, what kinds of triminoes exist? Let's draw them:



The first of these figures is often called a trimino-stick or a line, while the second is called a trimino-corner.

Tetromino — this is a figure consisting of four squares that can be cut out from a grid paper along the grid lines.

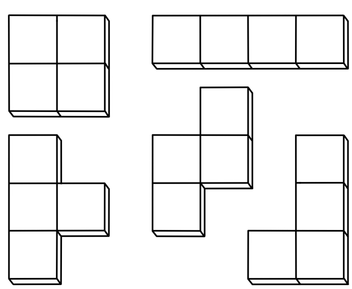
Famous tetrominoes include shapes like the "T," square, and rectangle.



These shapes are often used in the game "**Tetris**," where the goal is to move falling tetrominoes to fill horizontal lines. By the way, the word "**Tetris**" was derived from the words "**tetromino**" and "**tennis**."

Example 7.1. How many unique tetrominoes exist if, like dominoes and triminoes, shapes that can be rotated or reflected are considered the same?

Solution: All possible tetrominoes are shown in the diagram below. There are 5 unique tetrominoes.



□

There is a puzzle called **Polimino**. It's a fun game to play with friends. Let's explain how it works.

First, you and your opponent need to draw a 6×6 grid as the game board. Then, you and your opponent cross out a few forbidden squares on each other's boards. If the opponents are of different skill levels, you can simply cross out a different number of squares.

Here is an approximate difficulty guideline:

- On the easiest level, mark 3 **forbidden squares**

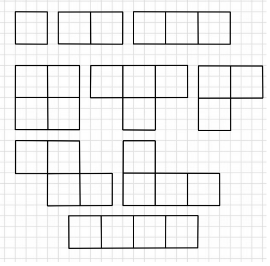
- On a medium level, cross out 4 **squares**.

- On a harder level, cross out 5 **squares**.

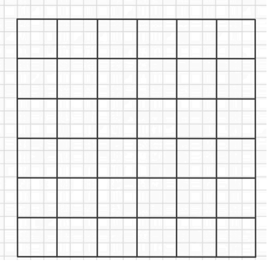
- On an advanced level, cross out 6 **squares**.

- If you're playing against a skilled adult, you can confidently cross out **7 squares**.

Next, you need to cut out a set of pieces: one single-square piece, one domino, both possible triminoes, and all five possible tetrominoes. Here's a template for cutting them out:



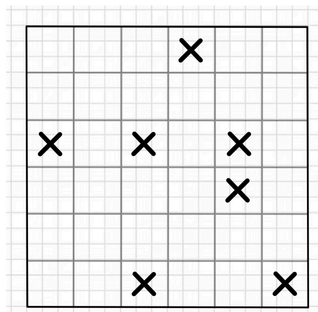
The easiest way to cut out these pieces is by using grid paper, where each cell is the size of a small 3×3 square. Then, use the same size for the game board:



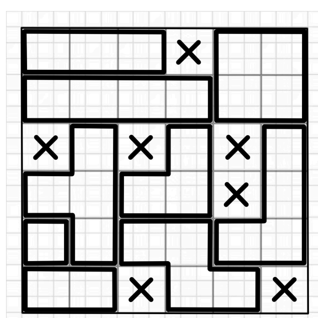
Once you've marked the forbidden squares and exchanged boards, the game begins. The winner is the first person to place all their pieces on the board without covering forbidden cells. You can rotate and flip the pieces.

The most interesting part of this puzzle is that it always has a solution!

Example 7.2. Place the polyomino pieces on the hardest level:



Solution: For example, the pieces could be placed like this:



□

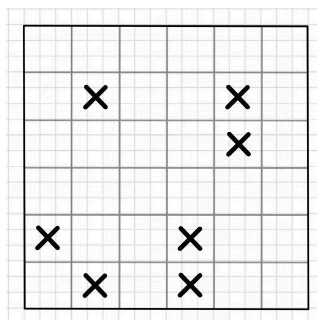
Let's take a short break from math and think about what the names of these games mean. It seems that **"mino"** refers to something like a square or cell. A single square can also be called a **monomino**, where the prefix **"mono"** means one.

The prefix **"do"** means two. The prefix **"tri"** is obvious—it means three. Similarly, the prefix **"tetra"** means four. You can even create a name for shapes with five squares: the prefix **"penta"** means five, giving us **pentomino**. And **"poly"** means many. All these prefixes come from Greek. Can you think of other words that use these prefixes?

Practice Problems

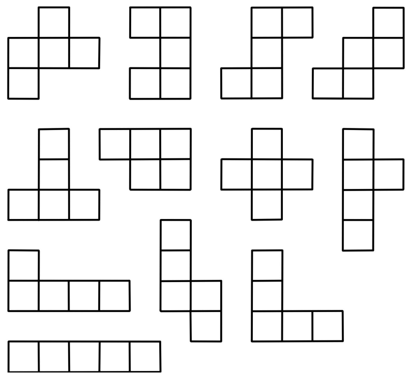
Problem 7.1. Pentomino — this is a figure consisting of 5 cells cut out from grid paper along the grid lines. Shapes that can be overlapped using rotations or reflections are considered the same. How many unique pentominoes exist?

Problem 7.2. Arrange the given set of polyomino pieces at the hardest game level:



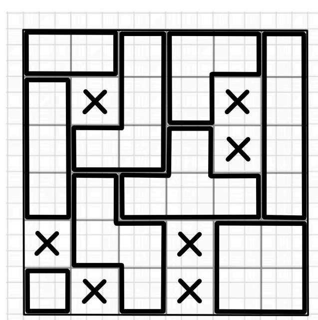
Solutions to Practice Problems

Solution to Problem 7.1: All possible pentominoes are shown in the diagram. There are a total of 12 types of pentominoes.



□

Solution to Problem 7.2: For example, the pieces could be arranged like this:

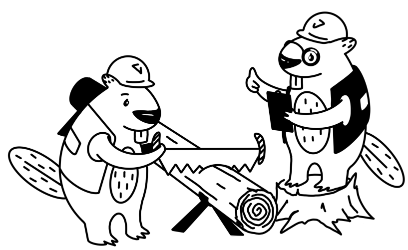


□

Logs and Splits

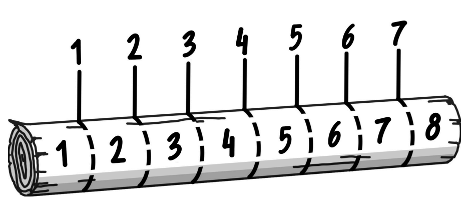
Theory and Practice

In math olympiads, especially for younger grades, problems often involve dividing logs (or other objects) into smaller parts. Let's solve a few such problems.



Example 8.1. Beavers were sawing a log. They made 7 cuts. How many pieces did they divide the log into?

Solution: Let's draw the log with cuts and count the resulting pieces:



We see that even though there were seven cuts, there are eight pieces! Let's remember this observation—it will come in handy later. □

Example 8.2. Max promised his mom he would read only 10 more pages of comics before studying math. After reading the 10th, 11th, 12th, ..., and 20th pages, he honestly started his lessons. However, his mom still accused him of breaking his promise. Was his mom right? Explain your answer.

Solution: At first glance, Max read $20 - 10 = 10$ pages. But there's a catch.

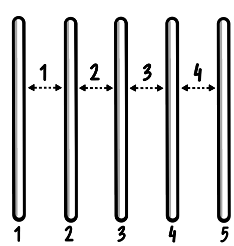
What pages did Max actually read? Let's list their numbers:

$$10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20.$$

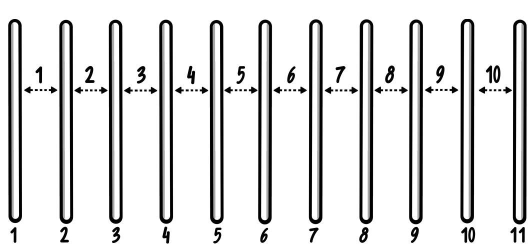
Counting these, we find there are eleven pages! So yes, Max did cheat. □

Example 8.3. Five sticks were placed on a desk. How many gaps are there between them? Answer the same question for 11 sticks and 32 sticks.

Solution. Let's draw five sticks and count the gaps between them:



We see that even though there are five sticks, there are only four gaps. What happens if we draw 11 sticks?



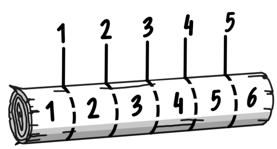
Here, there are only ten gaps.

Drawing thirty-two sticks would take too long. But that's okay—we've already identified a pattern! The number of gaps is always one less than the number of sticks. So for 32 sticks, there will be 31 gaps. □

Example 8.4. A log was split into 6 pieces. How many cuts were made?

Solution. Let's recall the first problem in this chapter. There, we made 7 cuts and got 8 pieces. Perhaps the logic is similar here: the number of pieces is always one more than the number of cuts. Let's test this.

Let's try making five cuts:



And indeed, we end up with 6 pieces. □

Now, let's solve a more challenging problem.

Example 8.5. To split a log, Leo marked it with blue and red lines. Cutting along the red lines results in 3 pieces. Cutting along the blue lines results in 4 pieces. How many pieces will there be if the log is cut along all the lines?

Solution: We'll solve this problem step by step. First, let's find the total number of marks. Cutting along the red lines gives 3 pieces. According to the logic we discovered earlier, there must have been two red marks.

Cutting along the blue lines gives 4 pieces, meaning there must have been three blue marks.

In total, there are $2 + 3 = 5$ marks.

And five cuts, as we know, result in six pieces. □

Practice Problems

Problem 8.1. One morning, Alice baked a round pie. How many cuts does Alice need to make so that her mom, dad, younger brother Max, grandma, grandpa, and herself each get a piece of the pie, and one piece remains for her older sister Beatrice?

Problem 8.2. First-grader Misha has 5 lessons at school on Tuesday. After each lesson (except the last one), there is a break. How many breaks does Misha have on Tuesday?

Problem 8.3. Apple trees are planted in a row along the boulevard, spaced one meter apart. What is the distance from the first apple tree to the tenth? And to the 12th?

Problem 8.4. The young beavers are sawing a log. They made 10 cuts. How many log pieces (known as "chunks") are there?

Problem 8.5. The numbers $15, 16, 17, \ldots, 30$ were written on the board. How many numbers are written in total?

Problem 8.6. A row of 8 apple trees was planted, spaced 2 meters apart. What is the distance between the first and the last apple tree?

Problem 8.7. There are 30 steps between the first and second floors. How many steps do you need to climb to get from the first floor to the third? How many steps are there to reach the fourth floor?

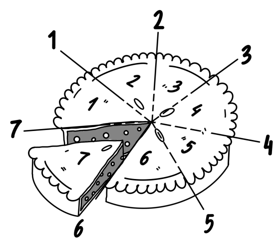
Solutions to Practice Problems

Solution to Problem 8.1: How many people need a piece of pie? Six and one more piece need to be left for Beatrice. So the pie must be divided into seven pieces. Would six cuts be enough?

No, there's a catch. Since the pie is round, we'll actually need seven cuts!

Although a round pie might resemble a very short log, there is a fundamental difference: we cut the pie from the center into triangular slices!

Here's an illustration:

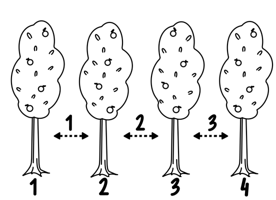


□

Solution to Problem 8.2: Misha will have breaks after the first lesson, the second lesson, the third lesson, and the fourth lesson, but no break after the fifth and final lesson. So, on Tuesday, Misha will have four breaks.

Let's generalize this. There is a break after each lesson except the last one, meaning the number of breaks is one less than the number of lessons. Doesn't this remind you of the gaps between trees? □

Solution to Problem 8.3: Let's draw a few apple trees to identify the pattern. To keep it simple, we'll draw four apple trees, for instance.



We see that there are three gaps between the four trees, which is one less than the number of trees. Therefore, the distance between the first tree and the tenth will be $10 - 1 = 9$ gaps, or 9 meters. Similarly, the distance between the first tree and the 12th will be $12 - 1 = 11$ gaps, or 11 meters.

Generalizing this, we observe that the number of gaps is one less than the number of trees. □

Solution to Problem 8.4: As we already know, the number of chunks will be one more than the number of cuts. So, there will be $10 + 1 = 11$ chunks. □

Solution to Problem 8.5: This problem resembles the one about comic books. In the comic book problem, our answer was one more than the calculated difference. What is the difference here? $30 - 15 = 15$. Thus, the total numbers written will be $15 + 1 = 16$. You can try listing all the numbers to verify this. □

Solution to Problem 8.6: As in the earlier problem about apple trees, the number of gaps is one less than the number of trees. So, there will be $8 - 1 = 7$ gaps, each 2 meters wide. If you know multiplication, you can simply calculate that the total distance will be $7 \times 2 = 14$ meters. If not, you can just add seven twos to get the same answer. □

Solution to Problem 8.7: To get from the first floor to the third, you need to climb two flights of stairs. First, you climb from the first floor to the second, and then from the second to the third. In total, you'll climb $30 + 30 = 60$ steps. Note that in this problem, we assume the number of steps per flight (the space between floors) is the same, although this is not explicitly stated. If you encounter a similar situation during an exam and have doubts, always ask the jury for clarification.

To go further to the fourth floor, you'll need to climb another 30 steps. In total, you'll climb $60 + 30 = 90$ steps. □

Logic: Knights and Liars

Theory and Practice

At last, we've reached a very unusual island inhabited by only two tribes: knights, who always tell the truth, and liars, who always lie.

This island of knights and liars appears frequently in Olympiad problems.

Example 9.1. You approached a local resident and asked about the color of the grass. They replied that it's red. Who are they?

Solution: Red grass? They definitely lied to us! Therefore, they must be a liar since knights always tell the truth.

Unless otherwise stated, the world in these problems is usually the same as ours. □

Example 9.2. You approached a local resident, and they told you their name is Vasya. Who are they?

Solution: Here, we can't determine for sure because we don't know their name! □

Example 9.3. You approached a local resident, and they said $2 + 5 = 7$. Who are they?

Solution: $2 + 5$ is indeed equal to 7. This means we heard the truth. Therefore, we are speaking to a knight because a liar would definitely lie. □

Example 9.4. You asked a knight who they are. What will they say?

Solution: A knight will obviously tell the truth. They will say, "I am a knight." □

Example 9.5. You asked a liar who they are. What will they say?

Solution: A liar will obviously want to lie. So, they won't admit to being a liar. Instead, they will say, "I am a knight." □

What did we observe? On this island, whoever we ask will claim they are knights! This is an important observation that will come in handy later.

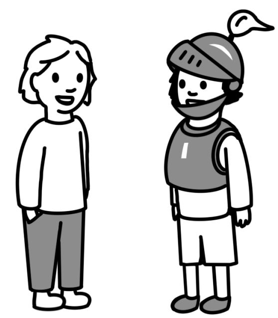
So, how can we determine whether someone is really a knight or not? We need to ask them a clever question. For example, we can simply ask if $2 + 2 = 4$. A knight will say that it's true, while a liar will say it's false.

Example 9.6. Come up with a statement that could be made by both a knight and a liar.

Solution: We've already encountered one such statement: "I am a knight." In fact, there are many such statements, such as, "My name is Jean." If the knight's name is

Jean, they could definitely say this. If the liar's name is not Jean, they might lie and say the same thing. □

Example 9.7. One day, islander Max said, "Yesterday, my neighbor told me that he is a liar"
Is Max a knight or a liar?



Solution: We know for sure that the neighbor couldn't have said they were a liar. After all, every islander insists they are a knight!
So, Max definitely lied. Therefore, Max is a liar. □

Let's solve a very challenging problem to conclude. If you don't understand it on the first try, that's okay; you can always come back to it later.

Example 9.8. You met a group of three islanders. Liar Leo, whom we know from a previous problem, said that both of his companions are knights. Who could they actually be?

Solution: What does the phrase "Both my companions are knights" mean? It's equivalent to saying, "All my companions are knights." But this was said by a liar, so it's not true. What does that mean?

From the chapter on negations, we remember that the negation of "all" is "not all." So, "Not all my companions are knights" means at least one of them is not a knight.

Thus, either both of Leo's companions are liars, or one companion is a liar and the other is a knight. □

Problem 9.1. Come up with a statement that only a knight can say.

Problem 9.2. Come up with a statement that only a liar can say.

Problem 9.3. Come up with a statement that neither a knight nor a liar can say.

Problem 9.4. What will a knight answer to the question, "Are you a liar?"

Problem 9.5. What will a liar answer to the question, "Are you a knight?"

Problem 9.6. On the island of knights and liars, you meet two friends and ask, "Who are you?" One of them says something, but you can't hear it well. His friend says, "He says he is a liar." Can you determine who the speaker is and who his friend is?

Problem 9.7. On the island of knights and liars, you meet two friends and ask, "Who are you?" One of them responds, "We are both liars." Can you determine who the speaker is and who his friend is?

Solutions to Practice Problems

Solution to Problem 9.1: Any truthful statement will work. For example, only a knight would say that $2 + 2 = 4$. □

Solution to Problem 9.2: Only a liar can say that $2 + 2 = 5$. □

Solution to Problem 9.3: For example, the phrase ”I am a liar.” If a liar says this, it would mean he told the truth, which he cannot do. And if a knight says it, then he would be lying, which is also impossible. □

Solution to Problem 9.4: The knight will answer ”No,” because he must tell the truth. □

Solution to Problem 9.5: The liar will answer ”Yes,” because he can only lie. □

Solution to Problem 9.6: This is almost the same problem as the one about the neighbor. No one on this island will ever say that they are a liar. Therefore, the friend is lying, and he is the liar. But what about the first one who answered? Unfortunately, we cannot determine to which tribe he belongs. □

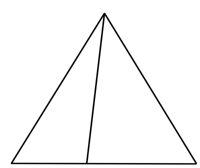
Solution to Problem 9.7: Let us assume that the speaker is a knight. Then, he must have told the truth. But that cannot be! This would mean that he is claiming to be a liar since he is one of the two friends. And we know that a knight would never call himself a liar.

Thus, the speaker must be a liar. What about the second one? If the second is also a liar, then both of them would indeed be liars. In that case, the phrase ”We are both liars” would be true. But a liar cannot tell the truth. Hence, the second one must be a knight. □

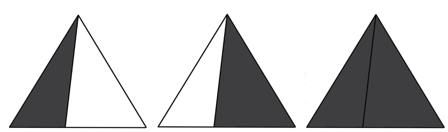
How many triangles are in the image?

Theory and Practice

How many triangles are in the image below?

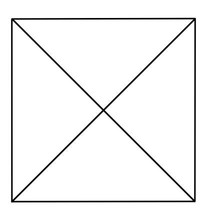


If you answered that there are 2, you fell into the trap. You probably didn't count the large third triangle made up of the two smaller ones.

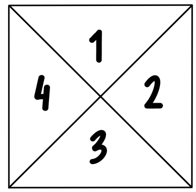


Let's try solving problems on this topic.

Example 10.1. How many triangles are in the following image?



Solution: The simplest way to account for all the triangles without missing any is to number the "pieces." For example, like this:



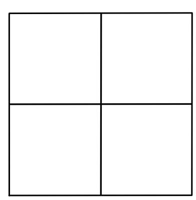
First, let's count the number of triangles consisting of a single piece. There are four: these are pieces 1, 2, 3, and 4.

How many triangles are made up of two pieces? There are also four: triangles made from pieces (1 and 2), (2 and 3), (3 and 4), and (4 and 1).

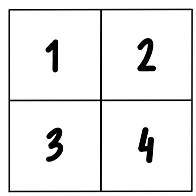
There are no triangles made from three or four pieces, which is clear from the diagram.

In total, we have $4 + 4 = 8$ triangles. □

Example 10.2. How many squares are in the following image?

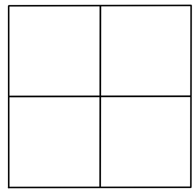


Solution: As before, let's number the pieces:

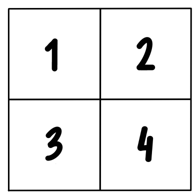


There are four squares consisting of a single piece. There are no squares made from two or three pieces (you might recall that 2 and 3 are not square numbers). But there is one square made of four pieces. So in total, there are five squares. □

Example 10.3. How many rectangles are in the following image?



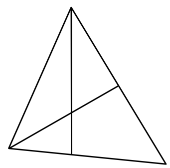
Solution: As before, let's number the pieces:



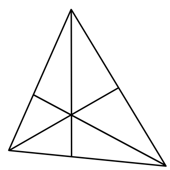
Here's a little trick. You may have already found all the rectangles made of two pieces: (1 and 2), (2 and 4), (4 and 3), and (3 and 1), and you might think that's all the rectangles in the image. But that's not true. In such problems, squares are also rectangles, just very symmetrical ones. So, we need to add the five squares we found in the previous problem. In total, we have $4 + 5 = 9$ rectangles. $\square$

Practice Problems

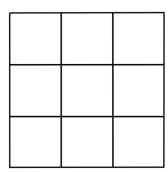
Problem 10.1. How many triangles are in the following image?



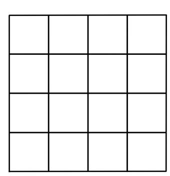
Problem 10.2. How many triangles are in the following image?



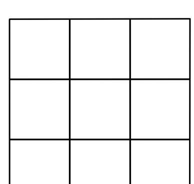
Problem 10.3. How many squares are in the following image?



Problem 10.4. How many squares are in the following image?

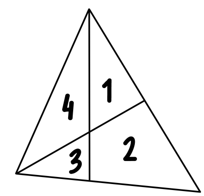


Problem 10.5. How many rectangles are in the following image?



Solutions to Practice Problems

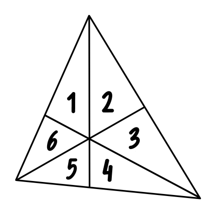
Solution to Problem 10.1: Number the pieces and count the triangles:



Of one piece. 1, 3, 4 – a total of three triangles.
Of two pieces. (1 and 2), (3 and 4), (3 and 2), (4 and 1) – a total of four triangles.
Of four pieces. (1, 2, 3, and 4) – a total of one triangle.

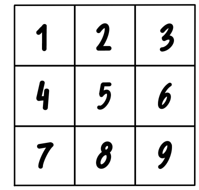
Thus, the total number of triangles is $3 + 4 + 1 = 8$. □

Solution to Problem 10.2: Number the pieces and count the triangles:



Of one piece. 1, 2, 3, 4, 5, and 6 – a total of 6 triangles.
Of two pieces. (1 and 6), (3 and 2), (5 and 4) – a total of 3 triangles.
Of three pieces. $(1, 2, 3)$, $(2, 3, 4)$, $(3, 4, 5)$, $(4, 5, 6)$, $(5, 6, 1)$, $(6, 1, 2)$ – a total of 6 triangles.
Of six pieces. (1, 2, 3, 4, 5, and 6) – a total of one triangle.
In total, there are $6 + 3 + 6 + 1 = 16$ triangles. □

Solution to Problem 10.3: Number the pieces and count the squares:



Of one piece. 1, 2, 3, 4, 5, 6, 7, 8, 9 – a total of 9 squares.
Of four pieces. $(1, 2, 4, 5)$, $(2, 3, 5, 6)$, $(4, 5, 7, 8)$, $(5, 6, 8, 9)$ – a total of 4 squares.
Of nine pieces. $(1, 2, 3, 4, 5, 6, 7, 8, 9)$ – a total of one square.
The total number of squares is $9 + 4 + 1 = 14$. □

Solution to Problem 10.4: Number the pieces and count the squares.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Of one piece. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16 – a total of 16 squares.

Of four pieces. $(1,2,5,6)$, $(2,3,6,7)$, $(3,4,7,8)$, $(5,6,9,10)$, $(6,7,10,11)$, $(7,8,11,12)$, $(9,10,13,14)$, $(10,11,14,15)$, $(11,12,15,16)$ – a total of 9 squares.

Of nine pieces. $(1,2,3,5,6,7,9,10,11)$, $(2,3,4,6,7,8,10,11,12)$, $(5,6,7,9,10,11,13,14,15)$, $(6,7,8,10,11,12,14,15,16)$ – a total of 4 squares.

Of sixteen pieces. The square consists of all pieces together – a total of 1 square.

The total number of squares is $16+9+4+1=30$. □

Solution to Problem 10.5: Number the pieces:

1	2	3
4	5	6
7	8	9

Of the previous problem, we know there are 14 squares in this figure. Now, count the rectangles that are not squares:

Of two pieces. Horizontal dominoes: $(1,2)$, $(2,3)$, $(4,5)$, $(5,6)$, $(7,8)$, $(8,9)$ – a total of 6 rectangles. Vertical dominoes: $(1,4)$, $(4,7)$, $(2,5)$, $(5,8)$, $(3,6)$, $(6,9)$ – another 6 rectangles. Thus, there are $6+6=12$ rectangles from 2 pieces.

Of three pieces. Horizontal trimino-“bars”: $(1,2,3)$, $(4,5,6)$, $(7,8,9)$ – 3 rectangles. Vertical trimino-“bars”: $(1,4,7)$, $(2,5,8)$, $(3,6,9)$ – another 3 rectangles. Total: $3+3=6$.

Of six pieces. Rectangles formed are: $(1,2,3,4,5,6)$, $(4,5,6,7,8,9)$, $(1,4,7,2,5,8)$, $(2,5,8,3,6,9)$ – 4 rectangles, two of size 3×2 and two of size 2×3.

In total, there are $6+12+6+4=28$ rectangles that are not squares. Adding the 14 squares gives $28+14=42$ rectangles in total. That's a lot, but don't worry, such problems are rare in real-life math competitions! $\square$

Siblings

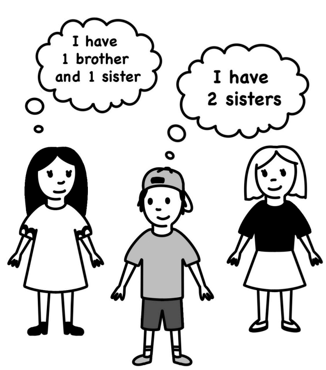
Theory and Practice

Now let's explore a very simple yet important topic. Problems like these are common in math olympiads and elementary school math clubs.

Example 11.1. The parents have 4 children. Each son has one sister. How many daughters are there in the family?

Solution: The key observation here is that if the family had more than one daughter, then each brother would have at least two sisters, which contradicts the condition. Therefore, there is exactly one daughter in the family. □

How might someone make a mistake here? By assuming that each brother has their own sister and concluding that there are 2 boys and 2 girls in the family.



Example 11.2. Anna has 4 brothers and 2 sisters. Ivan has 3 brothers and 3 sisters. Ivan is Anna's brother. How many boys and girls are there in the family?

Solution: To solve this problem, we only need the first part of the condition. If Anna has 4 brothers and 2 sisters, the parents already have 4 sons and 2 daughters. But we also need to include Anna herself, as she isn't her own sister, so we add one more girl. Therefore, the family has 4 boys and 3 girls.

Let's double-check using the second part of the condition. Similarly, Ivan has 3 brothers, 3 sisters, and himself. Again, we find that there are 4 boys and 3 girls in the family. □

Now, let's solve an interesting problem from a fifth-grade olympiad.

Example 11.3. In a family, each daughter has as many brothers as sisters, and each son has twice as many sisters as brothers. How many boys and girls are there in the family?

Solution: Let's begin with the first part of the condition: each daughter has as many brothers as sisters. We'll try different possibilities:

Case: Each daughter has one brother and one sister. In this case, the family has 2 girls and 1 boy. Then, each son has 0 brothers and 2 sisters — not a ratio of $2:1$.

Case: Each daughter has two brothers and two sisters. Here, the family has 3 girls and 2 boys. Each son would have 1 brother (the other boy) and 3 sisters. These numbers differ by $3:1$, not $2:1$.

Case: Each daughter has three brothers and three sisters. In this case, the family has 4 girls and 3 boys. Each son would have 2 brothers (the other boys) and 4 sisters. Since $2 \times 2 = 4$, the ratio works perfectly.

Thus, the correct answer is 4 girls and 3 boys in the family. □

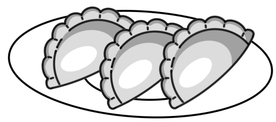
Practice

Problem 11.1. Mom buys one pie for each child. Each pie costs one tugrik. How many tugriks does Mom need to pay if two of her daughters have two brothers each, and one daughter has two sisters?

Problem 11.2. In one family, there are 5 children. It is known that each daughter has as many brothers as sisters. How many sons and daughters are in this family?

Problem 11.3. I have as many sisters as brothers. My sister has three times fewer sisters than brothers. How many of us are there?

Solutions to Practice



Solution to Problem 11.1: First, we need to determine how many children are in this family. One of the daughters has 2 brothers. This confirms there are two boys in the family. If another daughter has two sisters and is herself a girl, there must be three girls in the family. Thus, the family has $2+3=5$ children. Since each pie costs one tugrik, Mom needs to pay 5 tugriks. □

Solution to Problem 11.2: We know that each daughter has as many brothers as sisters. Let's test the possible cases.

Assume each sister has one brother and one sister. Then, the family would consist of two girls and one boy. This totals three children, not five as stated in the problem.

Assume each sister has two brothers and two sisters. In this case, the family would consist of three girls and two boys. This makes a total of five children, satisfying the problem's conditions.

We have found the correct answer: the family has two sons and three daughters. □

Solution to Problem 11.3: First, let's determine whether "I" in the problem is a son or a daughter. Noticing that "my" sister has three times fewer sisters than brothers, while "I" have an equal number of brothers and sisters, we can deduce that "I" and my sister must be of different genders. Therefore, "I" am a boy.

Since the boy has an equal number of brothers and sisters, we can systematically test the cases:

Assume the boy has one brother and one sister. The family would then have two boys and one girl. This would leave the sister with zero sisters and two brothers, which is not three times as many.

Assume the boy has two brothers and two sisters. In this case, the family would consist of three boys and two girls. The sister would then have three brothers and one sister, satisfying the condition that the number of brothers is three times the number of sisters.

We have found the correct answer: the family has five children. □

What's Heavier?

Theory and Practice

Example 12.1. What's heavier: 3 kilograms of cotton or 3 kilograms of nails?

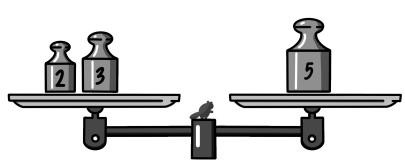
Solution: Many would confidently say that nails are obviously heavier. But that's not true, as both the cotton and the nails weigh exactly 3 kilograms! This means their weight is identical (though their density differs — a topic you'll learn about in upper grades). □

Of course, if these 3 kilograms were placed on you, the nails would seem heavier. That's your **subjective perception**. Nails are unpleasant and sharp, while cotton is soft. If someone were to put a fluffy kitten of greater weight on you, it might still seem lighter than nails. But in this chapter, we'll focus on **objective reality**.

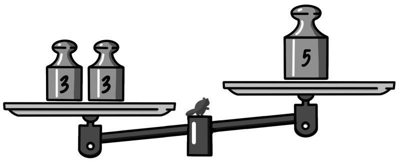
To solve weight-related problems, we need to understand what *balance scales* are and how they work.

Balance scales compare the weights placed on two pans. If the pans are at the same level, the weights are equal. Otherwise, the heavier side will be lower.

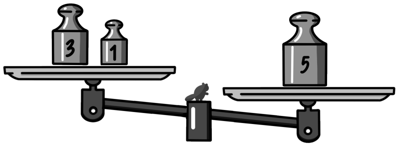
Here's an illustration.



These scales are balanced because $2 + 3 = 5$.



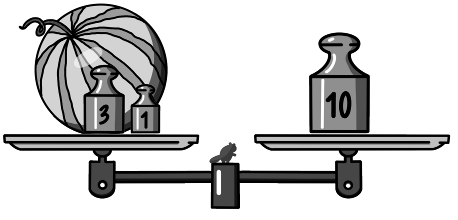
Here, the left pan is heavier because $3 + 3 = 6$, and 6 is greater than 5.



Here, the right pan is heavier because $3 + 1 = 4$, and 4 is less than 5.

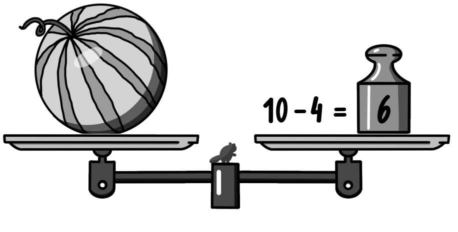
Now we're ready to solve a scale-related problem.

Example 12.2. What is the weight of the watermelon?



Solution: Next to the watermelon are weights of 3 and 1 kilograms. Altogether, $3 + 1 = 4$ kilograms are on the pan with the watermelon.

Now let's mentally remove 4 kilograms from each pan! What remains? * On the left pan, only the watermelon remains. On the right pan, we're left with $10 - 4 = 6$ kilograms.



Since the balance remains unchanged, the watermelon weighs 6 kilograms. □

In this problem, we used a clever trick. When solving scale-related problems, we can perform the same operation on both sides of the scales without changing their balance! For instance, if the scales are balanced and we add 1 kilogram to both sides, they'll remain balanced. If the right side was heavier and we remove 2 kilograms from each side, the right side will still be heavier.

Now let's tackle a trickier problem.

Example 12.3. A shopkeeper has three weights: 1 kg, 2 kg, and 4 kg. What weights of watermelons can he measure using only these weights and a balance scale, placing weights on just one pan?

Solution: The shopkeeper can measure watermelons weighing the same as each weight individually: 1 kg, 2 kg, and 4 kg. Additionally, using combinations of weights, he can measure:

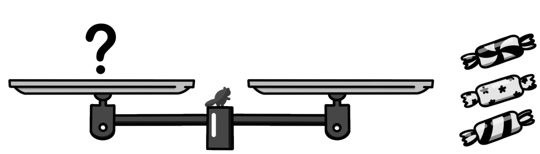
- $1 + 2 = 3$ (kg),
- $1 + 4 = 5$ (kg),
- $2 + 4 = 6$ (kg),
- $1 + 2 + 4 = 7$ (kg).

Thus, the shopkeeper can measure 1, 2, 3, 4, 5, 6, and 7 kilograms. □

If you're not in first grade anymore and have participated in math competitions, you've likely encountered more complex weighing problems.

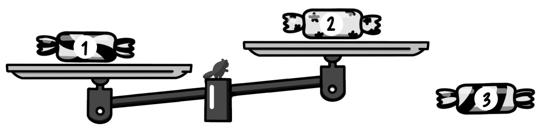
Here's a classic problem on this topic.

Example 12.4. You have 3 identical-looking candies, one of which is bad (lighter because it lacks filling). How can you identify the bad candy using a balance scale in just 1 weighing?

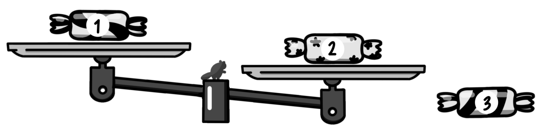


Solution: Let's number the candies and place candies 1 and 2 on opposite pans. There are three possible outcomes:

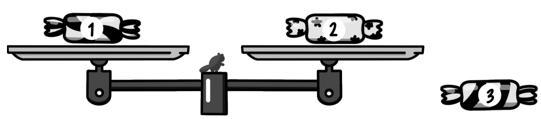
1. If the pan with candy 1 goes down, candy 1 is heavier than candy 2. Since only the bad candy is lighter, candy 2 must be the bad one.



2. Similarly, if the pan with candy 2 goes down, candy 1 must be the bad one.



3. If the scales balance, both candies weigh the same. Since there's exactly one bad candy, it must not be on the scales. Therefore, the bad candy is candy 3.



After considering all cases, we've completed the solution. Don't forget to account for all possibilities—even ones that seem obvious—since overlooking a single case could cost you points or even a complete solution in some competitions. □

Practice

Problem 12.1. A crane is lifting bags of material. The heavier the bag, the more fuel is consumed. Which will use more fuel: lifting 2 kilograms of iron or 2 kilograms of cotton?

Problem 12.2. Which is lighter: 4 kilograms of feathers or 2 kilograms of bricks?

Problem 12.3. A shopkeeper has three weights: 1 kg, 3 kg, and 9 kg. What weights can he measure if he places the weights only on one pan of the scales?

Problem 12.4. A shopkeeper has three weights: 1 kg, 3 kg, and 9 kg. What weights can he measure if he uses both pans of the scales?

Solutions to Practice Problems

Solution to Problem 12.1: Since both bags weigh 2 kilograms, the weight is identical. Therefore, the fuel consumption will be the same for both. $\square$

Solution to Problem 12.2: Four is greater than two. Therefore, 4 kilograms of feathers are heavier than 2 kilograms of bricks. $\square$

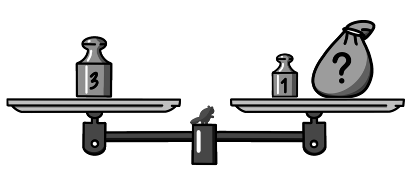
Solution to Problem 12.3: We list the possible weights:

- 1 (kg), 3 (kg), 9 (kg) — the weights of the individual weights.
- $1 + 3 = 4$ (kg),
- $1 + 9 = 10$ (kg),
- $3 + 9 = 12$ (kg),
- $1 + 3 + 9 = 13$ (kg).

Thus, the shopkeeper can measure the following weights: 1, 3, 4, 9, 10, 12, and 13 kilograms. $\square$

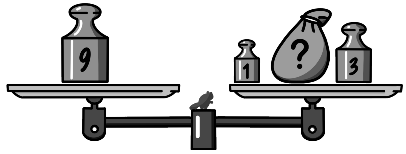
Solution to Problem 12.4: We already found that using only one pan, the shopkeeper can measure: 1, 3, 4, 9, 10, 12, and 13 kilograms.

Using both pans of the scales, additional weights can be measured. Denote the weight being measured as the bag.



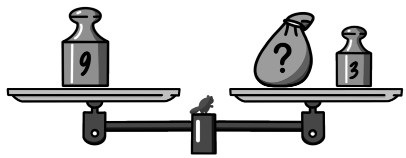
Here, the bag will weigh $3 - 1 = 2$ kilograms.

To measure 5 kilograms, arrange the weights as follows:



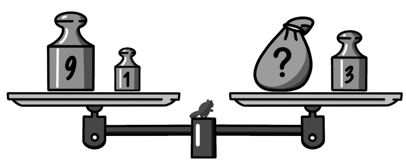
Indeed, the right side has weights totaling $3 + 1 = 4$ kilograms, so the bag weighs $9 - 4 = 5$ kilograms.

To measure 6 kilograms, arrange the weights like this:



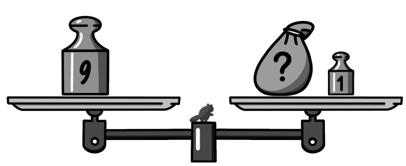
Here, $9 - 3 = 6$ kilograms.

To measure 7 kilograms, place the weights as follows:



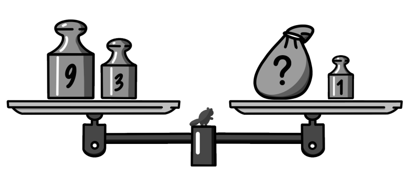
The left side weighs $9 + 1 = 10$ kilograms, so the bag weighs $10 - 3 = 7$ kilograms.

To measure 8 kilograms:



The bag weighs $9 - 1 = 8$ kilograms.

Finally, to measure 11 kilograms:



The left side weighs $3 + 9 = 12$ kilograms, so the bag weighs $12 - 1 = 11$ kilograms.

Thus, combined with the previously found weights, the shopkeeper can measure any weight, from 1 to 13 kilograms. $\square$

Find All the Numbers from 1 to . . . in the Picture

Theory and Practice

Mathematics isn't just about logic and patterns — it's also about attention to detail.

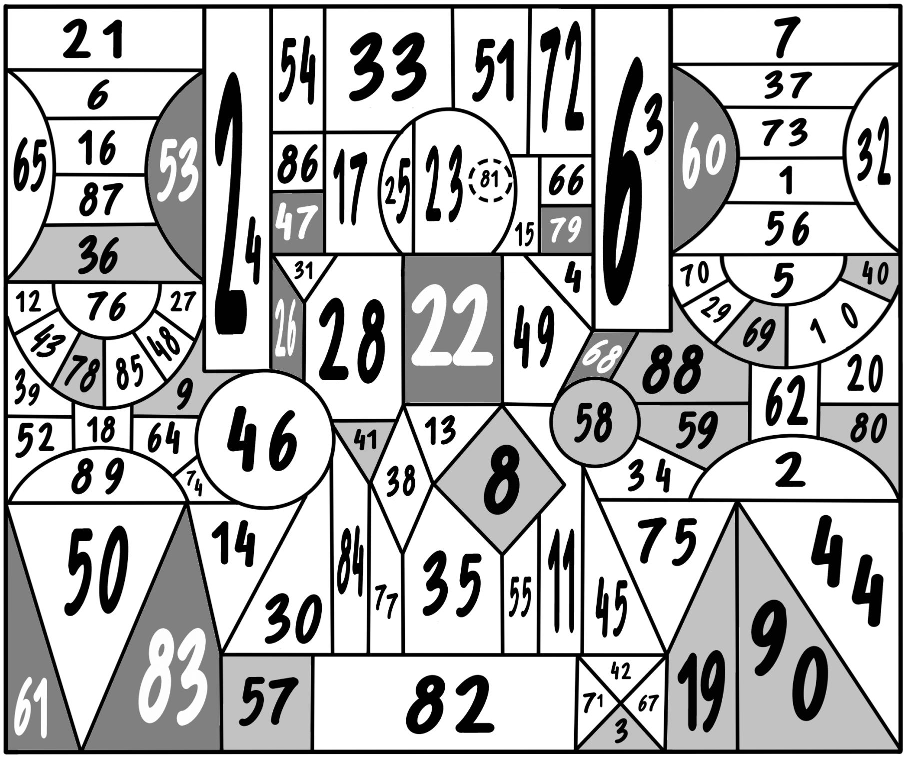
For example, try to find all the numbers from 1 to 25 in order on this picture. That means finding 1 first, then 2, and so on.

3	17	21	8	4
10	6	15	25	13
24	20	1	9	22
19	12	7	14	16
2	18	23	11	5

How long did it take you? Challenge your friends or even your parents to solve this puzzle. Who among you finishes fastest?

The difficulty of such puzzles can vary widely.

For instance, among adults in the Soviet Union, a similar version of this puzzle was very popular:



This table was frequently published in different sources, claiming that most people completed it in 5–10 minutes on average. A time under 10 minutes is considered excellent. 10 to 20 minutes is normal. And that's for adults! For kids, you can double those times — so if you finish it in 40 minutes, you're doing great!

You can even make this puzzle harder: for instance, by looking for the numbers in reverse order, from 90 down to 1. That's a surefire way to get even an adult's brain buzzing!

By the way, these types of puzzles are called **Schulte Tables**, and they're much more beneficial than they seem. For example, they help improve speed-reading skills — being able to read very quickly. They're also useful for adults, too. Research suggests that those who perform better on such tables tend to be more attentive while driving.

How should you approach solving these puzzles? The simplest way is to look for the numbers row by row, scanning the table in this order:

3	17	21	8	4
10	6	15	25	13
24	20	1	9	22
19	12	7	14	16
2	18	23	11	5

With practice, you'll gradually learn to see the entire table "as a whole," without scanning it line by line. This skill is called **peripheral vision**.

Peripheral vision refers to what you can see around you without turning your head or moving your eyes. If you imagine your field of vision as a circle, everything on the edges of that circle is part of your peripheral vision.

When you're looking straight ahead, for example, at a TV screen, you see everything in the center of your vision. At the same time, your peripheral vision allows you to notice objects or motion around you, even if you're not looking directly at them.

Peripheral vision is incredibly useful because it helps you spot important things around you, such as movement, people, or potential hazards, even while focusing on something else. It's like noticing toys on the floor while reading your favorite book.

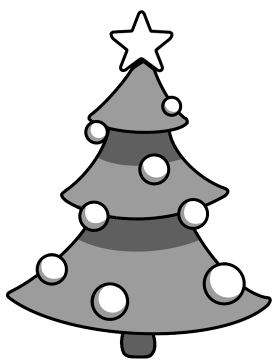
So, while keeping your eyes on something interesting in front of you, don't forget to use your peripheral vision to stay aware of what's happening around you!

Try practicing at home with the table below.

20	13	16	9	17
7	10	14	8	4
15	24	11	6	23
22	25	18	2	1
12	21	3	19	5

Reverse Moves

Theory and Practice



Example 14.1. The kids were decorating the Christmas tree. Suddenly, Vadik said, “Wow, we’ve already hung 12 ornaments!” How many ornaments were there in total if they still had 8 more to hang?

Solution: The initial number of ornaments can be calculated as the sum of those already on the tree and those remaining to be hung. Therefore, there were $12 + 8 = 20$ ornaments in total. $\square$

To solve some word problems, a useful method called “reverse moves” can be applied.

This method is particularly useful when unknown quantities change according to a rule, and the final result is known, but we need to find the starting value.

In some cases, by working backward through the sequence of operations, you can find the original value.

When working “in reverse,” it’s essential to remember that operations must be inverted. What does it mean to “invert”? Just like in the chapter “What’s Missing?”, you reverse addition with subtraction and vice versa. If you’ve forgotten, don’t worry—we’ll remind you of the basic rules in this chapter.

Let’s solve a problem.

Example 14.2. Vanya had some nuts. He gave 5 nuts to his dad. Then his sister gave him 3 more nuts. Vanya counted how many nuts he had and found that he now had 10. How many nuts did Vanya have at first?

Solution: Let's find out how many nuts Vanya had before his sister gave him 3 nuts. Since he had 10 nuts after her gift, we solve for the unknown in $\bigcirc + 3 = 10$. Remember that *first addend = sum - second addend*, so

$$\bigcirc = 10 - 3,$$

$$\bigcirc = 7.$$

Check: $7 + 3 = 10$. Thus, Vanya had 7 nuts before his sister gave him 3 nuts.

Now, let's find out how many nuts he had before giving 5 to his dad. If subtracting 5 from his original number of nuts leaves 7, we solve for the unknown in $\bigcirc - 5 = 7$. *To find the minuend, add the difference to the subtrahend.* Then:

$$\bigcirc = 7 + 5,$$

$$\bigcirc = 12.$$

Check: $12 - 5 = 7$, which is correct. So Vanya had 12 nuts before giving 5 to his dad.

Finally, let's verify the entire process. Vanya had 12 nuts. He gave 5 nuts to his dad, leaving $12 - 5 = 7$. Then his sister gave him 3 nuts, making $7 + 3 = 10$ nuts in total. Everything checks out, so the problem is solved correctly. □

Now, let's solve a more challenging problem that involves "doubling," which means adding a number to itself (or multiplying it by two).

Example 14.3. Misha thought of a number, added 1 to it, doubled it, and then added 1 again. The result was 9. What number did Misha think of?

Solution: Let's "unwind" the problem step by step. At the end, 1 was added to the number to make 9. Let's represent this number as $\bigcirc$. Then $\bigcirc + 1 = 9$. Before adding 1, the number was 8.

After doubling the number, it became 8. What number, when doubled, equals 8? Clearly, the number is 4.

At the very beginning, Misha added 1 to his number to get 4. Therefore, the number he thought of was 3.

Verification: $3 + 1 = 4$, $4 + 4 = 8$, $8 + 1 = 9$. Everything checks out. $\square$

Practice Problems

Problem 14.1. Anton received some candies for his birthday. His younger sister was upset because she didn't get any candies and started crying. To calm her down, Anton gave her 3 candies. As a result, he was left with 9 candies. How many candies did Anton receive as a gift?

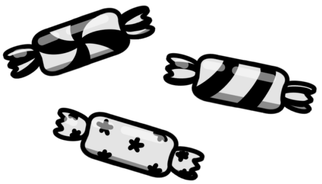
Problem 14.2. A mail carrier delivers letters from city A to city B. The carrier left city A, walked 4 kilometers, and stopped to rest. While resting, they calculated that there were still 5 kilometers left to city B. What is the distance between cities A and B?

Problem 14.3. Tanya bought some pens. Then her mother gave her the same number of pens as Tanya bought. After that, Tanya gave 3 pens to her best friend, and she was left with 7 pens. How many pens did Tanya buy initially?

Problem 14.4. Vasya had some candies. He ate half of them, and after that, he was left with only 3 candies. How many candies did Vasya have initially?

Problem 14.5. Petya thought of a number and then doubled it. After that, he subtracted 2 from the resulting number. Then he added 3 to it and got 7. What number did Petya think of?

Solutions to Practice Problems



Solution to Problem 14.1: Let's represent what happened using an equation: $\bigcirc - 3 = 9$. From this, we find that Anton originally had $9 + 3 = 12$ candies. $\square$

Solution to Problem 14.2: If the mail carrier walked 4 kilometers and still had 5 kilometers left to walk, then the total distance between cities A and B is $4 + 5 = 9$ kilometers. $\square$

Solution to Problem 14.3: In the end, Tanya gave 3 pens to her friend and was left with 7 pens. So, at that point, Tanya had $7 + 3 = 10$ pens.

Before this, her mother gave her as many pens as she already had. This means that doubling her initial pens gave her 10. Since $5 + 5 = 10$, Tanya initially bought 5 pens.

Verification: $5 + 5 = 10$, $10 - 3 = 7$. Everything checks out. $\square$

Solution to Problem 14.4: If Vasya ate half of his candies and was left with three candies, then he initially had twice that amount: $3 + 3 = 6$ candies. □

Solution to Problem 14.5: In the final step, 3 was added to the number to get 7. This means that before this addition, the number was $7 - 3 = 4$.

Then, 2 was subtracted to get 4. So, before the subtraction, the number was $4+2 = 6$.

Finally, the number was doubled to become 6. Since $3 + 3 = 6$, Petya must have initially thought of the number 3.

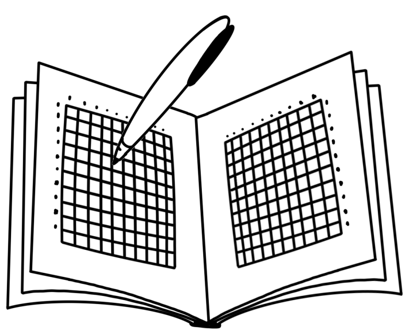
Verification: $3 + 3 = 6$, $6 - 2 = 4$, $4 + 3 = 7$. Everything checks out. □

Battleship

Theory and Practice

«**Battleship**» is a classic board game where players use tactical skills to sink their opponent's fleet. Here's how the game usually goes.

Each player gets a sheet of paper and a pen.



Each player draws two identical grids. One grid is used to place their ships, while the other is used to track their opponent's ships. Specific cells are typically identified using two coordinates: a letter (A, B, C, etc.) and a number (1, 2, 3, etc.). Unlike chess, Battleship often uses local alphabets instead of the Latin alphabet. In the classic version, the grid size is 10×10, meaning 10 cells by 10 cells.

Before the game starts, each player has two grids. On one grid, players place their ships. This is a strategically crucial step that can decide the entire game! Ships must be placed in such a way that makes it hard for the opponent to guess their positions. Ships come in various sizes: one-cell, two-cell, and so on. In the classic version, each player has four one-cell ships, three two-cell ships, two three-cell ships, and one four-cell battleship. Ships can be placed horizontally, vertically, or along the edges of the grid, as long as there is at least one cell of space between them. Ships cannot touch, even at the corners!

Players take turns firing shots at each other. A player selects a target cell on the opponent's grid by calling out its coordinates, combining the letter for the row and the number for the column. For example, E7. The opponent checks that cell on their grid. If it's empty, they declare "miss." An empty cell is one that isn't occupied by a ship or part of a ship. The player then marks that position on their opponent's grid as a miss and gives the turn to their opponent.

If the shot hits a ship, the opponent declares "hit" if part of the ship is damaged, or "sunk" if the entire ship is destroyed. The player marks this on their opponent's grid, usually by coloring the cell.

There are two versions of the game: one in which turns alternate and another in which a successful hit grants the player another turn.

Objective — Be the first to sink all of the opponent's ships.

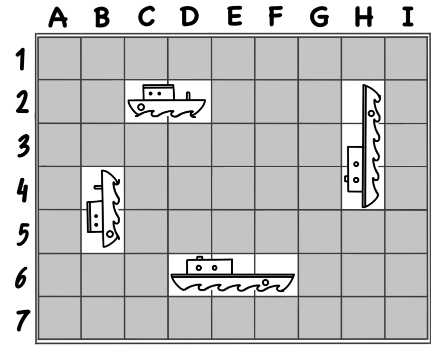
Once the game is over, players can discuss their strategies or start a new round.

«Battleship» is a great way to develop logical thinking, patience, and tactical skills. Plus, it's an exciting game—anyone can feel like a real admiral commanding their fleet!

By the way, ships of different sizes are often referred to by their cell count. For example, a *one-deck* ship is a one-cell ship, a *two-deck* ship is a two-cell ship, and so on.

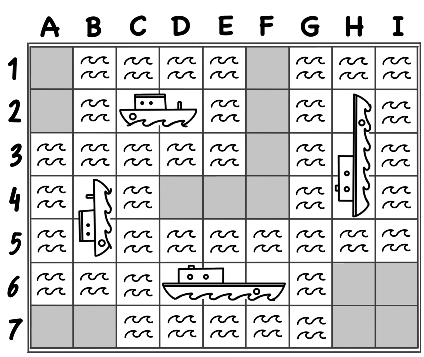
Understanding the logic of this game can significantly improve your chances of winning. Let's consider a sample problem from an Olympiad.

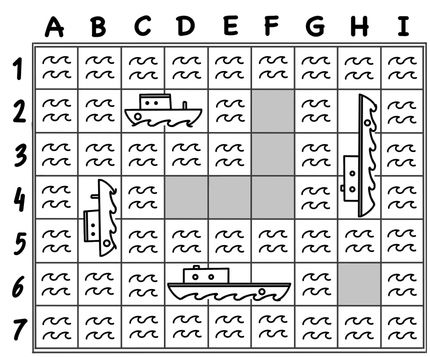
Example 15.1. The opponent's sunk ships are marked on the grid. It is known that a three-cell ship (three consecutive filled cells in a row or column) is still hidden somewhere, but not along the edges. Mark a cell where you should shoot to guarantee a hit. (Ships do not touch, even at the corners.)



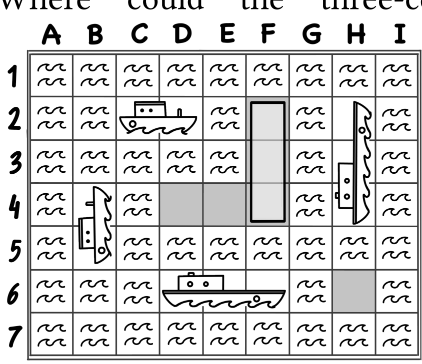
Solution: Since ships do not touch each other, even at the corners, we can immediately mark all cells around the sunk ships as empty. Let's show this on the left grid.

We also know that the ship is not along the edges, so all edge cells can also be marked as empty. Let's reflect this on the right grid.

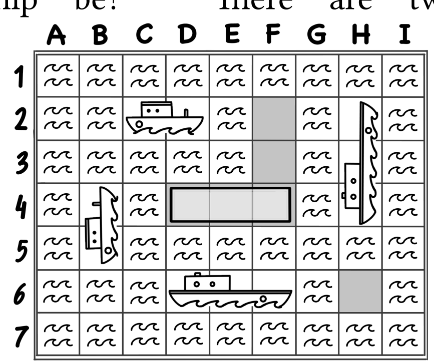




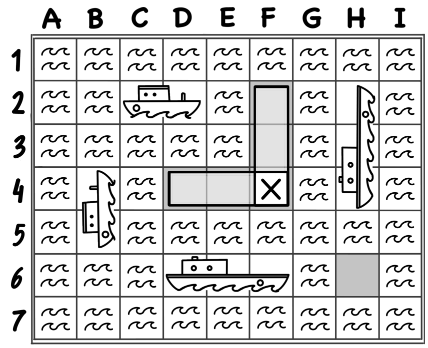
Where could the three-cell ship be? There are two possibilities:



or



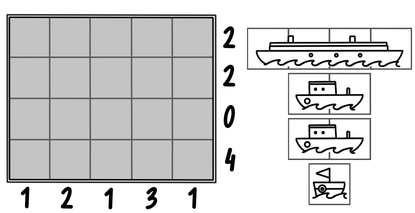
Let's overlay these possibilities:



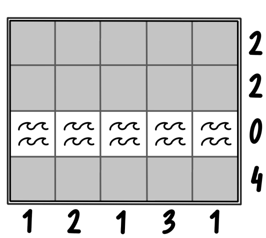
We can see that cell E4 is definitely occupied by the enemy ship, so that's where you should shoot. As you can see, logic is very helpful in games! □

There are also other puzzles involving Battleship. Let's solve one of them.

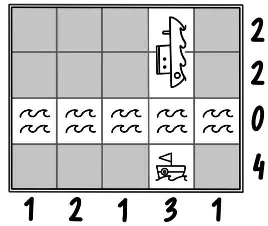
Example 15.2. Place the given set of ships on the grid so that they don't touch each other, even at the corners. The numbers outside the grid indicate the number of filled cells in the corresponding row or column.



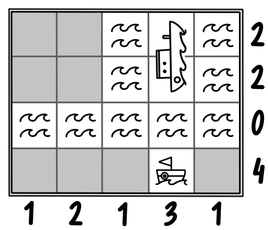
Solution: First, notice that there are no ship cells in the third row, so we can fill it with water.



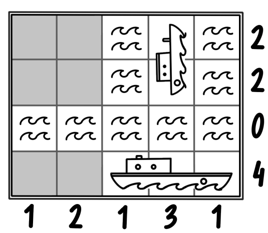
Now observe that the fourth column must contain three ship cells. Since there are only three remaining cells, we must fill them all.



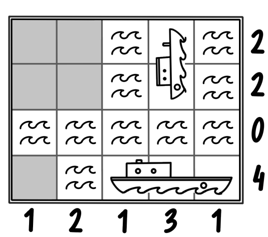
We found a two-cell ship in the fourth column. Mark water around it.



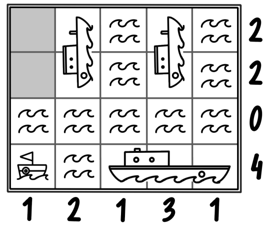
Now notice that the third and fifth columns each have one unmarked cell, which corresponds to a single ship cell. Mark this on the grid.



We found a ship at the bottom with at least three cells. Since we need to place one single-cell ship, two two-cell ships, and one three-cell ship, this must be the three-cell ship, with water around it.



Now it's easy to see that the second column has a two-cell ship, and the bottom row has a single-cell ship.

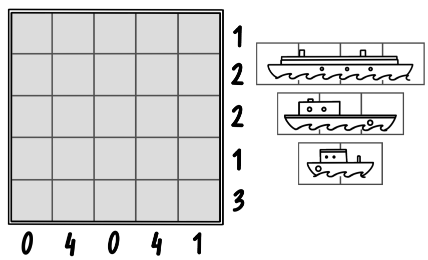


We solved the puzzle—well done! □

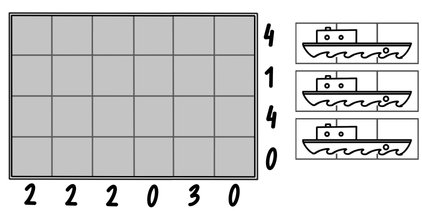
We encourage you to play Battleship with friends a few times. Once you become a true admiral, try solving the proposed puzzles.

Practice Problems

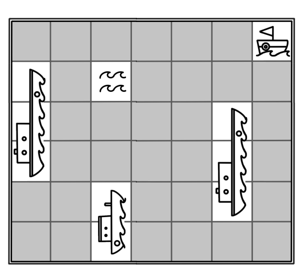
Problem 15.1. Place the given set of ships on the grid so that they do not touch each other, even at the corners. The numbers outside the grid indicate the number of ship cells in the corresponding row or column.



Problem 15.2. Place the given set of ships on the grid so that they do not touch each other, even at the corners. The numbers outside the grid indicate the number of ship cells in the corresponding row or column.

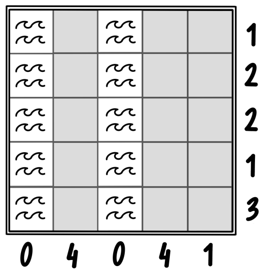


Problem 15.3. On the Battleship grid, the opponent's sunk ships are marked. It is known that a three-cell ship (three consecutive filled cells in a row or column) is still hidden somewhere, but not along the edges. One cell where a "miss" has already been recorded is also marked. Identify one cell where you need to shoot to guarantee a hit on the three-cell ship. (Ships do not touch each other, even at the corners.)

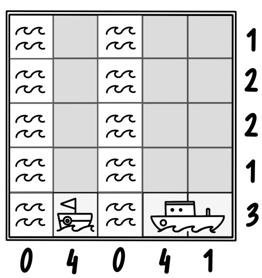


Solutions to Practice Problems

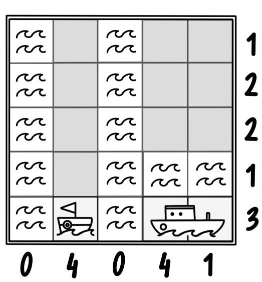
Solution to Problem 15.1: We need to place a four-cell ship, a three-cell ship, and a two-cell ship. First, mark all cells in the first and third olumns as empty since there are no ship cells there:



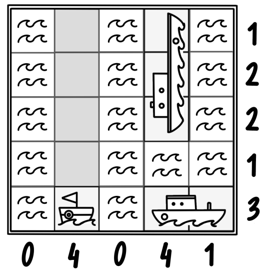
In the bottom row, there should be three ship cells, and only three potential cells remain. Therefore, all these cells must be occupied by ship parts:



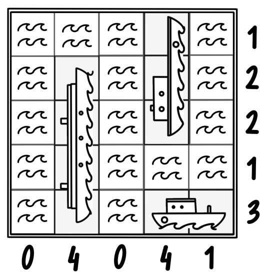
Notice that the bottom-right corner already contains a two-cell ship. Thus, the cells around it must be empty.



The fourth column should contain four ship cells. One cell is already occupied, and one is confirmed as empty, so the remaining cells must all contain ship parts. We have identified a three-cell ship! Mark it and the surrounding water:



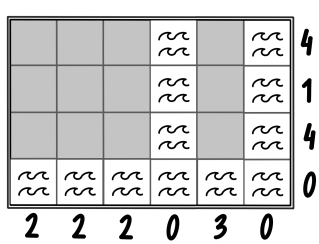
Finally, we need to place the four-cell ship. This is straightforward since we know one of its cells is already marked at B1. We can "extend" the marked one-cell ship.



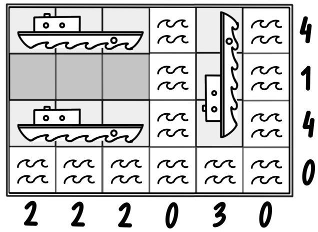
All ships have been placed. Don't forget to double-check that the numbers match for all rows and columns! □

Solution to Problem 15.2: Here, we need to place three three-cell ships.

First, mark all empty cells in rows and columns with zeros:

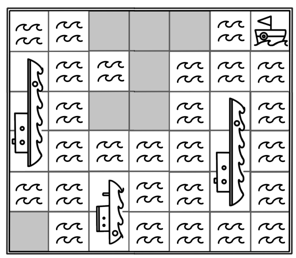


Next, mark the three ship cells in the fifth column. Now, it's easy to mark the three missing ship cells in the first and third rows:

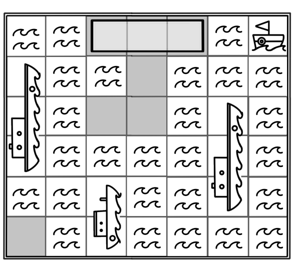


Great, we've found all three three-cell ships! Be sure to double-check that everything matches.

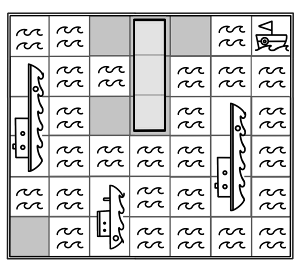
Solution to Problem 15.3: First, mark all cells adjacent to the known ships as empty.



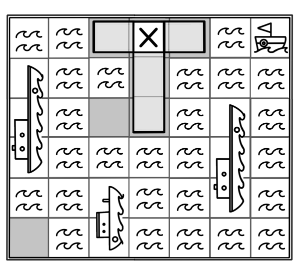
Where could the three-cell ship be? There are two possibilities:



or



By combining both possibilities, we find the common cell where we need to shoot:



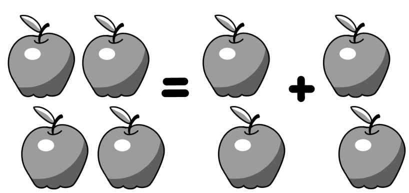
Even Numbers

Theory and Practice

Suppose you have four apples, and you want to share them equally with your mom. How many apples will each of you get?

It's easy to see that each of you gets 2. But what if you have three apples? In this case, you can't divide them equally without cutting one apple.

If the number of apples can be shared equally between two people, the number is called **even**, and if not, the number is called **odd**.



Let's practice a bit.

Are the following numbers even or odd: 6, 7, 8, 9, 10?

- 6 — the number is even because $3 + 3 = 6$.
- 7 — the number is odd. If each person gets three apples, that's $3 + 3 = 6$, which is less than seven, leaving one apple left over. If you give each person four apples, that's $4 + 4 = 8$, which is too many, as there aren't enough apples.
- 8 — the number is even because $4 + 4 = 8$.

- 9 — the number is odd because $4+4=8$, which is too few, and $5+5=10$, which is too many.
- 10 — the number is even because $5+5=10$.

We noticed an important pattern: even and odd numbers **alternate**, meaning an even number is always followed by an odd number, and vice versa.

Let's practice identifying even and odd numbers. In the following problems, you need to place even numbers in triangles $\triangle$, and odd numbers in circles $\bigcirc$, so the equations work.

Example 16.1.

$$\triangle + \bigcirc + \bigcirc = 16.$$

Solution. For example, one possible solution is $10+3+3=16$. This isn't the only way, so try to find other possibilities. $\square$

Example 16.2.

$$\triangle + \triangle + \bigcirc = 13.$$

Solution: For example, one solution is $10+2+1=13$. $\square$

Let's add another condition: now, all the numbers you use must be different.

Place different even numbers in the triangles:

Example 16.3.

$$\triangle + \triangle + \triangle = 16.$$

Solution: For example, $10+4+2=16$. $\square$

Example 16.4.

$$\triangle + \triangle + \triangle = 20.$$

Solution: For example, $10 + 8 + 2 = 20$. □

Example 16.5.

$$\triangle - \triangle + \triangle = 14.$$

Solution: For example, $10 - 2 + 6 = 14$. □

As we solved these problems, you might have noticed a few important properties:

Even number + Even number = Even number.

Even number + Odd number = Odd number.

Odd number + Odd number = Even number.

Try verifying these properties yourself. These are very important rules, and you'll encounter them many times in higher grades.

Practice Problems

Insert different even numbers into triangles $\triangle$, and different odd numbers into circles $\bigcirc$, so that the equations become correct.

Problem 16.1.

$$\triangle + \triangle + \bigcirc = 19.$$

Problem 16.2.

$$\bigcirc - \triangle + \bigcirc = 20.$$

Problem 16.3.

$$\triangle + \triangle - \bigcirc = 21.$$

Problem 16.4.

$$\bigcirc - \bigcirc + \bigcirc = 19.$$

Problem 16.5.

$$\triangle - \triangle - \bigcirc = 19.$$

Solutions to Practice Problems

Solution to Problem 16.1: For example, $10 + 8 + 1 = 19$. ☐

Solution to Problem 16.2: For example, $21 - 2 + 1 = 20$. ☐

Solution to Problem 16.3: For example, $20 + 2 - 1 = 21$. ☐

Solution to Problem 16.4: For example, $21 - 3 + 1 = 19$. ☐

Solution to Problem 16.5: For example, $22 - 2 - 1 = 19$. ☐

Sudoku

Theory and Practice

Sometimes, in Olympiads, there are problems where no proof is required; you simply need to provide a solution. For example, coloring a figure with different colors. Beginners often enjoy these types of problems because there's no need for formal proofs, though finding a solution is not always easy. In 1852, Scottish physicist Francis Guthrie, while working on a map of England's counties, observed that only four colors were needed to color it. His brother mentioned this observation to O. De Morgan, a well-known mathematician, and later A. Cayley formulated the famous "four-color problem" in 1878: Can any world map be colored with just four colors such that no adjacent regions share the same color? Despite its apparent simplicity, proving this problem was not easy. It was only solved in 1976 by Kenneth Appel and Wolfgang Haken, with a significant portion of the calculations performed by a computer. The proof was so extensive (hundreds of pages and thousands of diagrams) that very few mathematicians have been able to read it in its entirety.

One of the classic non-Olympiad topics involving coloring is Sudoku. Let's explain what Sudoku is for those unfamiliar with it.

When playing **Sudoku**, the game board is a 9×9 square divided into smaller 3×3 squares. Thus, the entire game board consists of 81 unit cells. At the start of the game, some cells already contain numbers (from 1 to 9), known as hints. The player's task is to fill the empty cells with digits from 1 to 9 so that each digit appears exactly once in every row, column, and 3×3 square. The classic Sudoku is played with nine colors (digits), though there are variations with larger or smaller grids, such as 4×4 grids divided into 2×2 squares. Let's explore the simplest methods for solving such problems.

Example 17.1. Solve the Sudoku.

1			
	2		
		3	1
			4

Solution: Let's label the cells on the board, like in chess or Battleship, for an easier explanation of the solution process.

	a	b	c	d
4	1			
3		2		
2			3	1
1				4

Notice that in the bottom-right square, three of the four cells are already filled, so the remaining cell $C1$ can only contain the number 2. Let's mark this.

	a	b	c	d
4	1			
3		2		
2			3	1
1			2	4

Now consider the bottom-left square. It must contain the number 1. This digit cannot be in column A, as 1 is already in cell $A4$. Nor can it be in row 2, as 1 is already in cell $D2$. Thus, 1 can only be in cell $B1$. Similarly, in the top-right square, 1 can only be in cell $C3$.

	a	b	c	d
4	1			
3		2	1	
2			3	1
1		1	2	4

Next, let's determine where 2 can go in the bottom-left square. It can only be in cell $A2$. Now, in row 2, three numbers are already filled, so the remaining cell $B2$ must contain 4. This leaves the number 3 for cell $A1$ in the bottom-left square. Consequently, the only remaining number for column A is 4, which must be placed in cell $A3$.

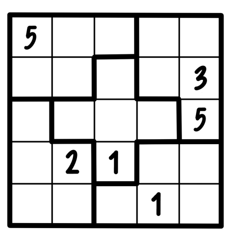
	a	b	c	d
4	1			
3	4	2	1	
2	2	4	3	1
1	3	1	2	4

By repeating these steps, we arrive at the solution. The completed Sudoku solves the problem.

	a	b	c	d
4	1	3	4	2
3	4	2	1	3
2	2	4	3	1
1	3	1	2	4

□

Example 17.2. Solve the Sudoku:



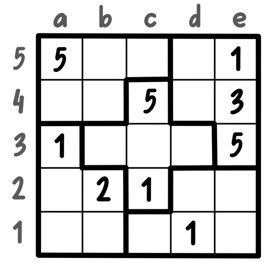
Solution: Once again, we'll name the cells like in chess.

Where can the number 5 go in the central square? Only in cell $C4$, as 5 is already in row 3.

Where can the number 1 go in the top-right square? Only in cell $E5$, as 1 is already in column D.

Where can 1 go in the bottom-left square? Only in cell $A3$, as 1 is already in rows 1 and 2.

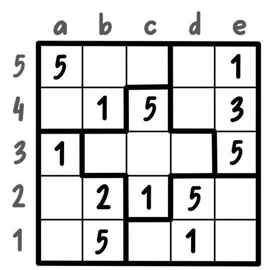
Let's mark this:



Where can the number 5 go in the bottom-left square? Only in cell $B1$, as 5 is already in column A.

Where can 5 go in the bottom-right square? Only in cell $D2$, as 5 is already in columns C and E.

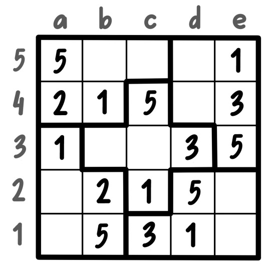
Where can 1 go in the top-left square? Only in cell $B4$, as 1 is already in row 5 and column A.



Where can 3 go in the bottom-right square? Only in cell $C1$, as 3 is already in column E.

Where can 3 go in column D? Only in cell $D3$, as the upper cells belong to the top-right square, which already contains 3.

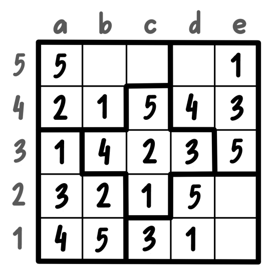
Where can 2 go in column A? Only in cell $A4$, as the lower cells belong to the bottom-left square, which already contains 2.



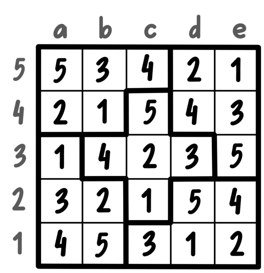
Row 4 now has only the number 4 remaining.

The central square has only 2 and 4 left. The number 2 cannot be in cell $B3$, as 2 is already in column B. Thus, $B3$ contains 4, and $C3$ contains 2.

The bottom-left square has 3 and 4 left. The number 3 cannot be in cell $A1$, as 3 is already in row 1. Thus, $A1$ contains 4, and $A2$ contains 3.



Column B now has only 3 left. Column C has only 4 left. Column D has only 2 left. Row 2 has only 4 left. Row 1 has only 2 left. The problem is solved!

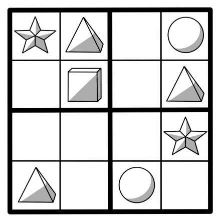


□

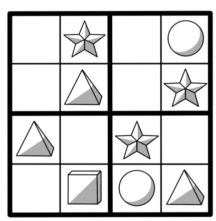
Practice Problems

In all the proposed tasks, you are required to solve Sudoku puzzles. In each task, you need to fill in the cells with symbols (pictures, numbers from 1 to 4, numbers from 1 to 5, numbers from 1 to 6, or numbers from 1 to 7) so that in each row, each column, and each highlighted figure (square in the first five tasks), all the symbols are unique.

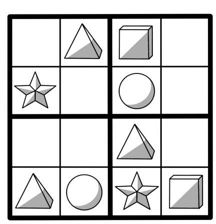
Problem 17.1.



Problem 17.2.



Problem 17.3.



Problem 17.4.

2			1
	4	3	
	2	1	
3			4

Problem 17.5.

4		2	1
			3
	2	3	

Problem 17.6.

4			3
	2		
		3	
			1

Problem 17.7.

1			
			3
			4
2	4		

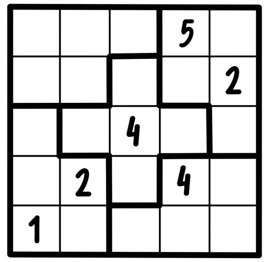
Problem 17.8.

3			1
4			2
	3	1	
	4	2	

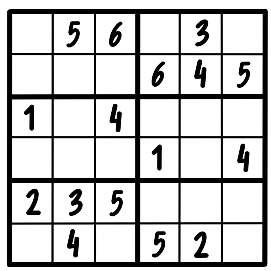
Problem 17.9.

				1
	3			
		3		
			2	
5		1		

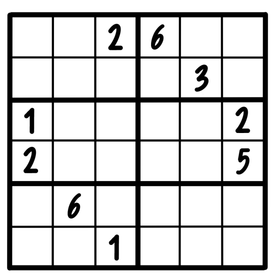
Problem 17.10.



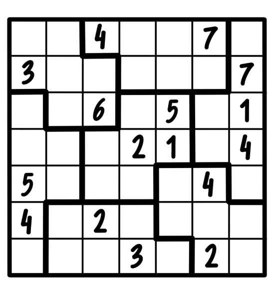
Problem 17.11.



Problem 17.12.



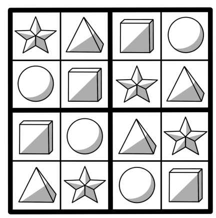
Problem 17.13.



Solutions to Practice Problems

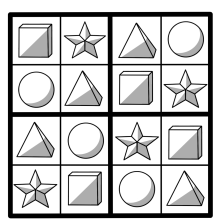
To solve these problems, we use exactly the methods and reasoning discussed in this chapter. Therefore, we only provide the answers here.

Solution to Problem 17.1:



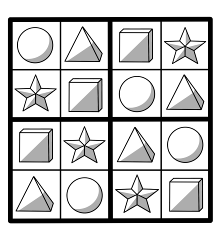
□

Solution to Problem 17.2:



□

Solution to Problem 17.3:



□

Solution to Problem 17.4:

2	3	4	1
1	4	3	2
4	2	1	3
3	1	2	4

Solution to Problem 17.5:

4	3	2	1
2	1	4	3
3	4	1	2
1	2	3	4

Solution to Problem 17.6:

4	1	2	3
3	2	1	4
1	4	3	2
2	3	4	1

Solution to Problem 17.7:

1	3	4	2
4	2	1	3
3	1	2	4
2	4	3	1

Solution to Problem 17.8:

3	2	4	1
4	1	3	2
2	3	1	4
1	4	2	3

□

Solution to Problem 17.9:

2	4	5	3	1
1	3	2	5	4
4	5	3	1	2
3	1	4	2	5
5	2	1	4	3

□

Solution to Problem 17.10:

4	5	1	5	3
2	3	5	1	2
3	1	4	2	4
5	2	3	4	1
1	4	2	3	5

□

Solution to Problem 17.11:

4	5	6	2	3	1
3	1	2	6	4	5
1	6	4	3	5	2
5	2	3	1	6	4
2	3	5	4	1	6
6	4	1	5	2	3

□

Solution to Problem 17.12:

4	3	2	6	5	1
6	1	5	2	3	4
1	5	3	4	6	2
2	4	6	3	1	5
5	6	4	1	2	3
3	2	1	5	4	6

□

Solution to Problem 17.13:

1	2	4	5	3	7	6
3	4	5	1	2	6	7
2	7	6	4	5	3	1
6	3	7	2	1	5	4
5	1	3	6	7	4	2
4	5	2	7	6	1	3
7	6	1	3	4	2	5

□

Combinatorics

Theory and Practice

How many ways are there to choose three apples from a basket? How many options are there for a school schedule? How many ways can a class be divided into two teams? These types of questions are addressed by **combinatorics**, which you will study in depth in middle school. In combinatorial problems, the main question is: How many combinations can we form from a set of objects that satisfy specific conditions?

In simple cases, we can list all the combinations we need and count them. However, listing should never be done randomly! Proper enumeration involves listing numbers in ascending order or words in alphabetical order; this ensures that no options are missed and avoids duplicate entries.

Example 18.1. How many two-digit numbers can be made from the digits $1, 2, 3$?

Solution: List the numbers in ascending order:

$$11, 12, 13, 21, 22, 23, 31, 32, 33.$$

There are 9 numbers in total. □

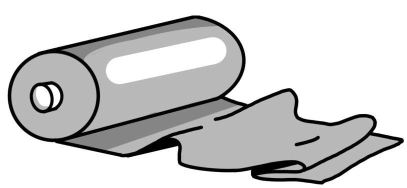
Example 18.2. By tomorrow, you need to prepare math, Russian, and geography homework (the order doesn't matter). How many ways can the homework be prepared?

Solution: Let's encode our subjects with letters: **M** for math, **R** for Russian, and **G** for geography. For instance, **MRG** represents preparing math first, followed by Russian, and then geography. List the options in alphabetical order: **GMR**, **GRM**, **MGR**, **MRG**, **RGM**, **RMG**. There are 6 options in total. □

Example 18.3. How many ways can two red balls and two blue balls be arranged in a row? The balls are identical except for their color, so we cannot distinguish between balls of the same color.

Solution: Let's represent red balls as **R** and blue balls as **B**. List the arrangements in alphabetical order: **RRBB**, **RBRB**, **RBBR**, **BRRB**, **BRBR**, **BBRR**. There are 6 options in total. □

Example 18.4. A store sells white, black, and green fabric. You need to buy fabric in two different colors. How many options are there to choose from?



Solution: There are 3 options: white and black, white and green, and black and green. □

Example 18.5. How many ways can three different flowers be placed into two vases?

Solution: Label the flowers as 1, 2, and 3. The left of the comma represents one vase, and the right represents the other:

1. nothing, 123;
2. $1, 23$;
3. $2, 13$;
4. $3, 12$;
5. $12, 3$;
6. $13, 2$;
7. $23, 1$;
8. 123, nothing.

There are 8 options in total. □

Example 18.6. How many two-digit numbers less than 30 can be formed using the digits $1, 2, 4$?

Solution: List the suitable numbers in ascending order:

$$11, 12, 14, 21, 22, 24.$$

There are 6 numbers in total. □

Example 18.7. List all triples of natural numbers that sum up to 10. Triples that differ only in the order of numbers are considered identical.

Solution: We will arrange the numbers in **non-decreasing order**, meaning each subsequent number is not smaller than the previous one. For example, $1, 1, 2$ is a valid non-decreasing triple.

What are the triples where the first number is 1? If the first number is 1, the other two numbers must sum to $10 - 1 = 9$. These pairs are $1 + 8$, $2 + 7$, $3 + 6$, $4 + 5$. The pair $5 + 4$ is not valid because 5 is greater than 4. Hence, the triples starting with 1 are: $1 + 1 + 8$, $1 + 2 + 7$, $1 + 3 + 6$, $1 + 4 + 5$.

Similarly, triples starting with 2 are $2 + 2 + 6$, $2 + 3 + 5$, $2 + 4 + 4$.

There is only one triple starting with 3: $3 + 3 + 4$.

No triples can start with 4 or higher because $4 + 4 + 4 = 12$, which exceeds 10.

In total, there are $4 + 3 + 1 = 8$ triples, and we have listed them all. $\square$

Interestingly, *brute-force methods* are widely used in computer science and related fields. For instance, a password is considered cryptographically secure if there is no faster method to "crack" it than brute force. Unfortunately for hackers, many people still choose very simple passwords.

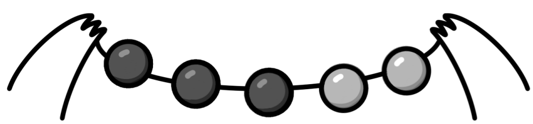
Practice Problems

Problem 18.1. List all pairs of natural numbers that sum to 15. Pairs that differ only in the order of numbers are considered identical.

Problem 18.2. List all triples of distinct natural numbers that sum to 15. Triples that differ only in the order of numbers are considered identical.

Problem 18.3. List all quartets of distinct natural numbers that sum to 15. Quartets that differ only in the order of numbers are considered identical.

Problem 18.4. Sonya strung 3 blue beads and 2 red beads in a row. How many different arrangements can she make? List all options.



Problem 18.5. Vasya has 4 pieces of fabric in different colors — red, blue, green, and white. He wants to make a four-colored flag with all horizontal stripes. How many different flags can he make if the top stripe must be red?

Solutions to Practice Problems

Solution to Problem 18.1: Arrange the numbers in non-decreasing order. We get the following pairs: $1+14=15$, $2+13=15$, $3+12=15$, $4+11=15$, $5+10=15$, $6+9=15$, $7+8=15$. Thus, there are seven possible pairs. $\square$

Solution to Problem 18.2: To list triples of distinct numbers, arrange them in ascending order (each subsequent number must be greater than the previous one).

- Triples starting with 1: $1+2+12$, $1+3+11$, $1+4+10$, $1+5+9$, $1+6+8$.
- Triples starting with 2: $2+3+10$, $2+4+9$, $2+5+8$, $2+6+7$.
- Triples starting with 3: $3+4+8$, $3+5+7$.
- Triples starting with 4: $4+5+6$.

Thus, there are $5+4+2+1=12$ possible triples. $\square$

Solution to Problem 18.3: For quartets, we organize the numbers by their smallest value and arrange them in ascending order.

Quartets starting with 1: The remaining three numbers must sum to $15-1=14$.

- If the second number is 2: $1+2+3+9$, $1+2+4+8$, $1+2+5+7$.
- If the second number is 3: $1+3+4+7$, $1+3+5+6$.

Quartets starting with 2: The remaining three numbers must sum to $15-2=13$.

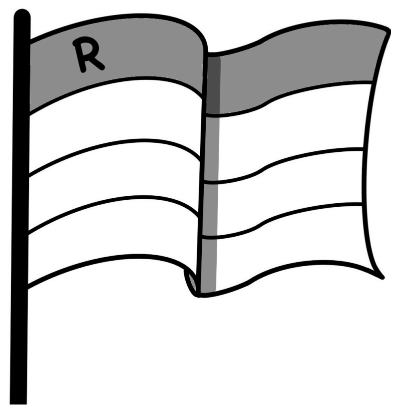
- If the second number is 3: $2+3+4+6$.

Thus, there are 6 quartets: $1+2+3+9$, $1+2+4+8$, $1+2+5+7$, $1+3+4+7$, $1+3+5+6$, $2+3+4+6$. $\square$

Solution to Problem 18.4: Let **R** represent a red bead and **B** represent a blue bead.

List the arrangements in alphabetical order: **RRBBB**, **RBRBB**, **RBBRB**, **RBBBR**, **BRRBB**, **BRBRB**, **BRBBR**, **BBRRB**, **BBRBR**, **BBBRR**. Thus, there are 10 arrangements. □

Solution to Problem 18.5: Since the top stripe must be red, the arrangement starts as follows:



Now, we arrange the remaining stripes (blue, green, white), denoted as **B**, **G**, and **W**, in alphabetical order: **RWGB**, **RWBG**, **RGWB**, **RGBW**, **RBWG**, **RBGW**. Thus, there are 6 possible flags. □

Good bye!

In the conclusion of my book for beginner Olympiad enthusiasts, I would like to emphasize the importance and diversity of the topics covered. These mathematical problems and puzzles contribute to the development of logical thinking, a creative approach to problem-solving, and the formation of skills in combinatorics and abstract thinking.

A wide range of topics, from simple numeric riddles to complex logic and combinatorics problems, has helped you gain confidence in solving mathematical challenges and introduced you to the fascinating world of mathematics.

We hope that with the advent of this book, preparing for Olympiads has become not only a useful mental exercise but also an exciting journey.

As you may have noticed, we did not require you to know mathematics beyond the first grade. Yet, we solved problems even for higher grades, such as sixth grade! And if you've made it to the end of our book, then you are a true hero!

I sincerely shake your hand. I am proud of you!

See you soon!